Deconfined criticality and gapless $\mathbb{Z}_2$ spin liquids in the square lattice antiferromagnet

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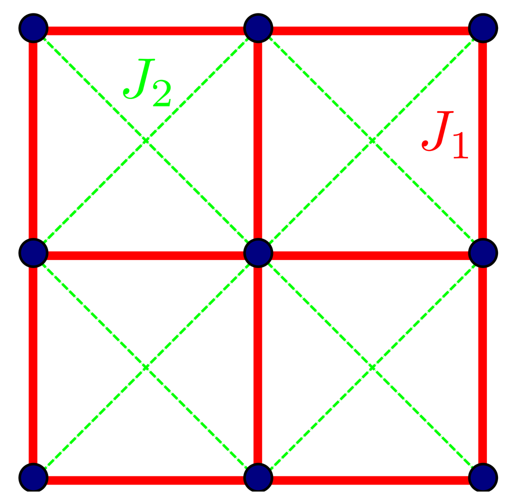
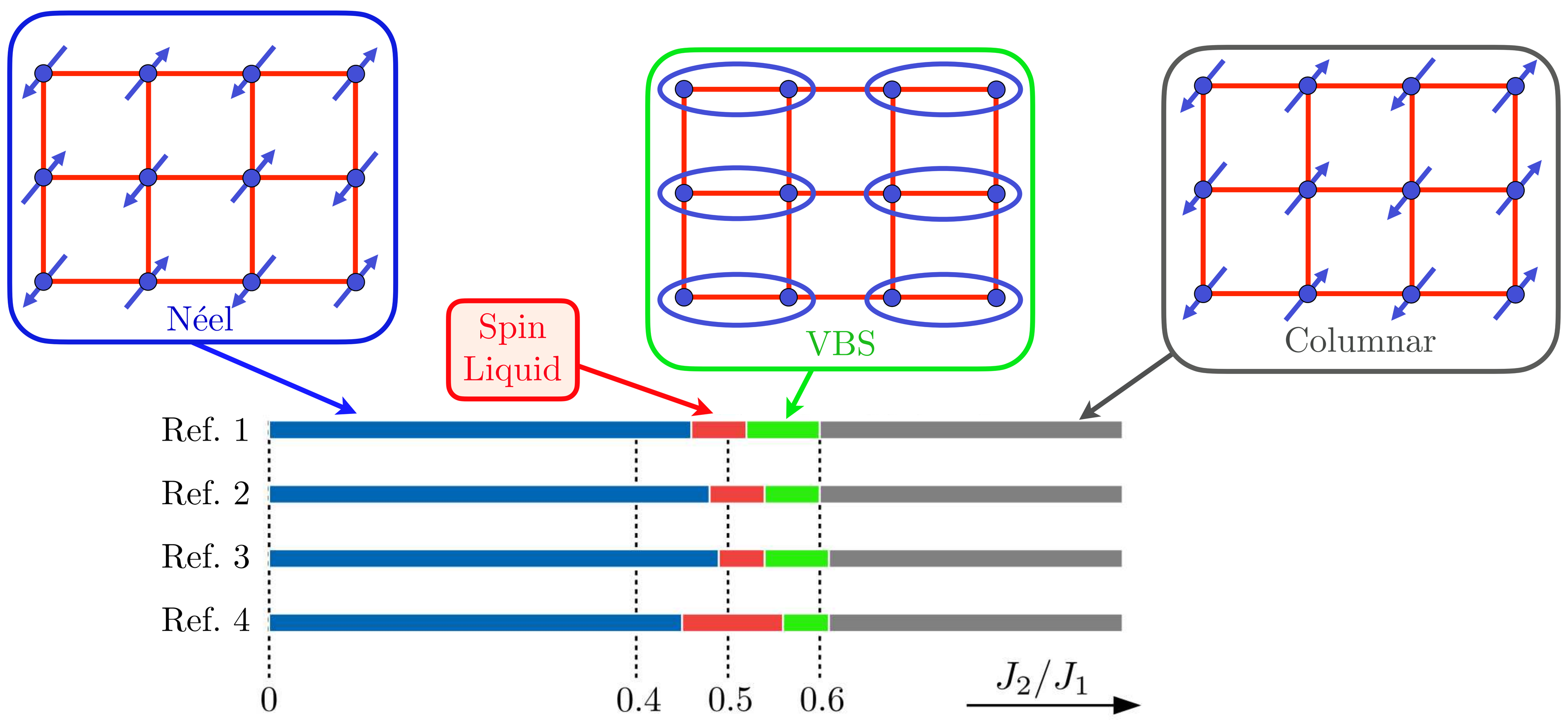
Numerical evidence for a gapless $\mathbb{Z}_2$ spin liquid in the J_1-J_2 model

Our model of interest is the spin-1/2 antiferromagnetic J_1-J_2 model on the square lattice,

$$H = J_1 \sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle ij\rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Although the nature of the ground state in the regime of strong frustration ($J_2/J_1 \approx 0.5$) has been an open question for decades, a recent set of numerical studies in [1, 2, 3, 4] give evidence for a stable Z2 spin liquid phase bordering Néel and VBS orders.



Modeling spin liquid transitions via Higgs condensation

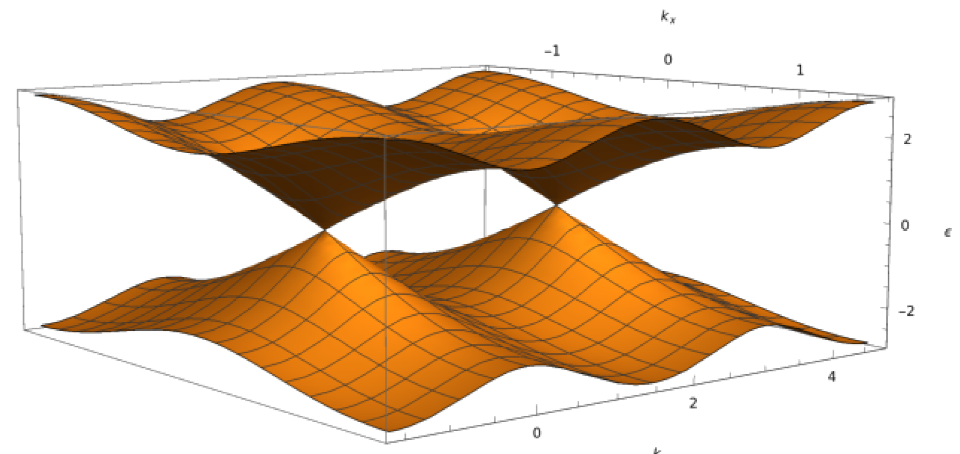
We propose that transitions from a gapless $\mathbb{Z}_2$ spin liquid conjecture to various ordered phases (Néel, VBS) can be described starting from the continuum theory of the π-flux phase. This is an SU(2) gauge theory that is conjectured to be unstable to Néel or VBS ordering [5]. Our theory includes additional Higgs fields, which can condense to break the SU(2) gauge symmetry to $\mathbb{Z}_2$. Proximate to both these phases is an additional U(1) phase - the staggered flux phase. This phase is also unstable to Néel or VBS ordering via monopole proliferation [6, 7].

Continuum model for transitions

To model these transitions, we use the parton construction to describe different spin liquid phases, writing our spin operators as fermionic spinons:

$$\mathbf{S}_i = \sum_{\alpha\beta} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}\,.$$

This rewriting introduces an SU(2) gauge redundancy. At mean-field level, different phases can be described by different free spinon Hamiltonians, with additional corrections coming from gauge fluctuations [8]. The gapless $\mathbb{Z}_2$ spin liquid we are interested in has four Dirac points, and the low-energy theory is of four massless Dirac fermions coupled to a $\mathbb{Z}_2$ gauge field.

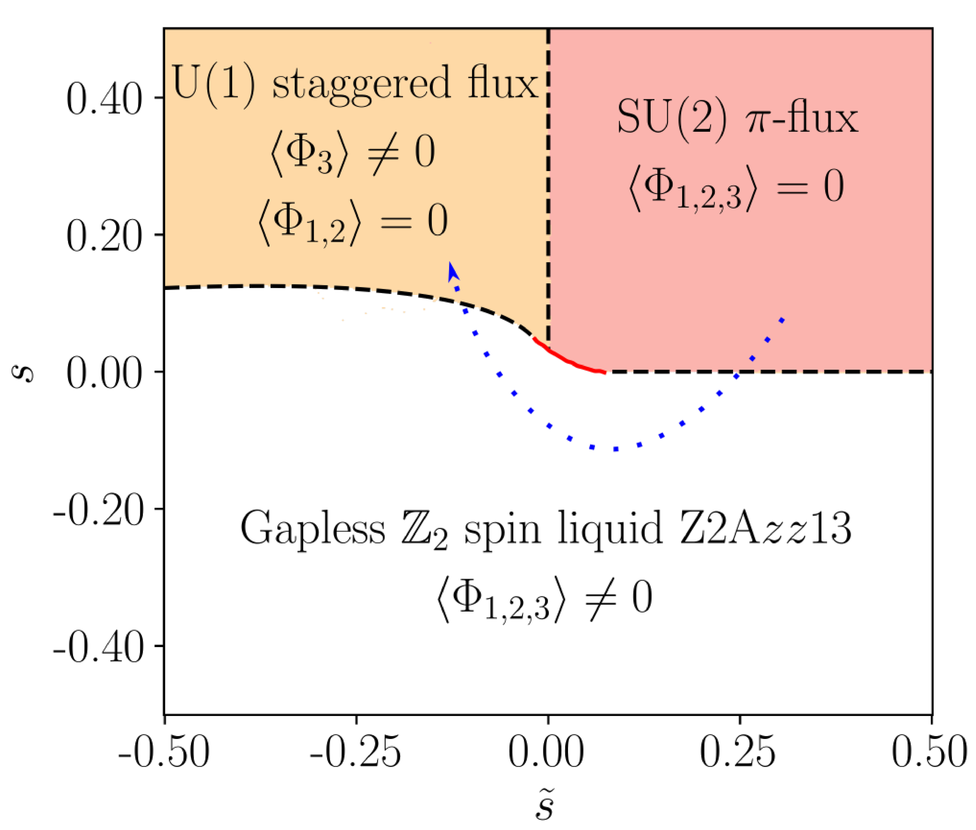


Lagrangian and mean-field diagram

We propose a Lagrangian that describes a gapless $\mathbb{Z}_2$ spin liquid, along with instabilities to proximate SU(2) (π-flux) and U(1) (staggered flux) spin liquids. This Lagrangian contains four Dirac fermions ψ, with matrices σ (τ) acting on gauge (valley) space, coupled to three adjoint Higgs fields $\Phi_{1,2,3}$ and an SU(2) gauge field A_μ. Explicitly, this Lagrangian is

$$\mathcal{L} = i\bar{\psi}\not{D}\psi + \Phi_1^a\bar{\psi}\sigma^a\mu^z\gamma^x\psi + \Phi_2^a\bar{\psi}\sigma^a\mu^x\gamma^y\psi + \Phi_3^a\bar{\psi}\sigma^a\mu^y(\gamma^y i\partial_x + \gamma^x i\partial_y)\psi + V(\Phi)$$

The microscopic symmetries of the square lattice dictate the form of the Yukawa couplings and constrain the form of $V(\Phi)$, leading to the mean-field phase diagram shown. Our conjectured trajectory of the J_1/J_2 model, with the U(1) and SU(2) phases being proxies for Néel or VBS ordering, is shown in blue. Both of the two critical theories are studied in a $1/N_f$ expansion, where $4N_f$ is the number of fermions.

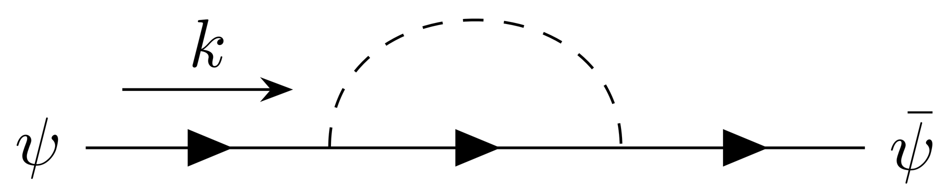


$\mathrm{SU}(2) \to \mathbb{Z}_2$ transition: emergent subsystem symmetries and UV/IR mixing

This transition [9] is driven by the simultaneous condensation of the $\Phi_{1,2}$ fields. In a $1/N_f$ expansion, the physics of this phase transition is controlled by emergent subsystem symmetries,

$$\psi \to e^{i\sigma^a\mu^z f_a(x)}\psi \quad \Phi_1^a \to \Phi_1^a + \partial_x f_a(x)$$
$$\psi \to e^{i\sigma^a\mu^x g_a(y)}\psi \quad \Phi_2^a \to \Phi_2^a + \partial_y g_a(y)$$

This symmetry is only respected by the Yukawa couplings and the bare fermion propagator, but other terms are irrelevant to all orders in a $1/N_f$ expansion. This leads to divergences in momentum space at generic momenta (neither UV nor IR), and necessitates the inclusion of "dangerously irrelevant" terms which break this subsystem symmetry and cures the divergence. This leads to logarithm-squared corrections in quantities such as the fermion self-energy,



$$\Sigma(k) \approx -\frac{3}{\pi^2 N_f}\not{k}\ln^2(K|k|)\,,$$

where K is the coefficient of the dangerously irrelevant term. This leads to correlation functions that are neither power-law nor exponential,

$$G(r) \sim r^\beta \exp\left[-\eta \ln^2(r)\right]$$

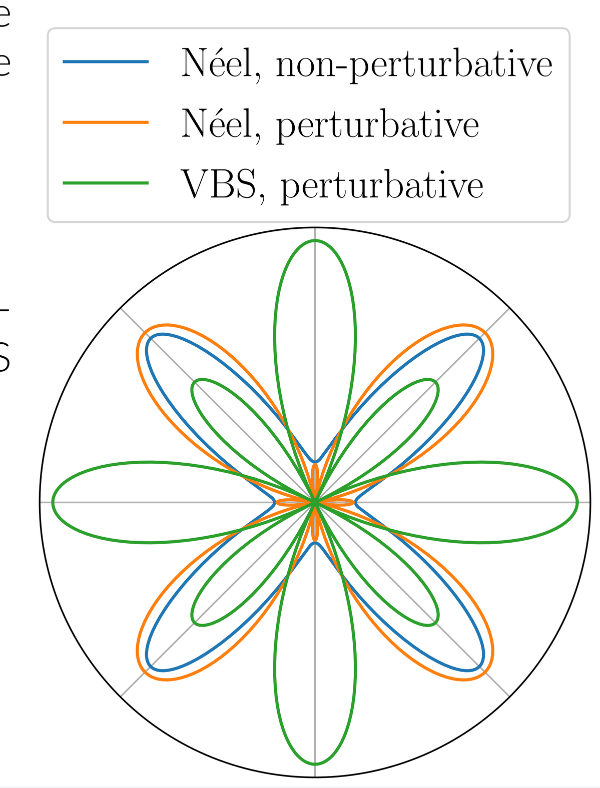
with β a *non-universal* exponent and

$$\eta_{\text{VBS}} = \frac{6}{\pi^2 N_f} + \mathcal{O}\left(N_f^{-2}\right) \qquad \eta_{\text{Néel}} = \frac{12}{\pi^2 N_f} + \mathcal{O}\left(N_f^{-2}\right)$$

$\mathrm{U}(1) \to \mathbb{Z}_2$ transition: anisotropic deconfined criticality

The $\mathrm{U}(1) \to \mathbb{Z}_2$ transition [10] is described by the condensation of a single complex Higgs field. At leading order in a $1/N_f$ expansion, there is a single stable fixed point with an anisotropic fermion dispersion relation,

$$\mathcal{L}_\psi = i\bar{\psi}\not{\partial}\psi + \Phi_c\bar{\psi}\sigma^z\mu^y(\gamma^x i\partial_y + \gamma^y i\partial_y)\psi\,,$$
$$\Phi_c = 0.46 + \mathcal{O}\left(N_f^{-1}\right).$$



This anisotropy leads to non-trivial angular profiles in the Néel and VBS correlation functions. The fixed point values of the anomalous Néel and VBS correlation functions, as well as the dynamical critical exponent z, are

$$\eta_{\text{VBS}} = \frac{0.06}{N_f} + \mathcal{O}\left(N_f^{-2}\right)$$
$$\eta_{\text{Néel}} = -\frac{0.01}{N_f} + \mathcal{O}\left(N_f^{-2}\right)$$
$$z = 1 + \frac{0.225}{N_f} + \mathcal{O}\left(N_f^{-2}\right)$$

Predictions of our critical theories

We present two possible critical theories describing instabilities of a gapless $\mathbb{Z}_2$ spin liquid to either Néel or VBS order. Both yield predictions that may be tested in numerics, including

- A violation of Lorentz invariance ($z \neq 1$)
- For the SU(2) transition, violation of power-law scaling for observables - this may be reflected in a drift in fitted critical exponents as a function of system size or irrelevant perturbations, reminiscent of a weakly first-order transition
- For the U(1) transition, a violation of SO(2) spatial rotation symmetry leading to non-trivial angular profiles of the Néel and VBS correlation functions

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