# Numerical study of the random tJ model with all-to-all interactions

Henry Shackleton

Harvard University

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# Hamiltonian and Phase Diagram

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

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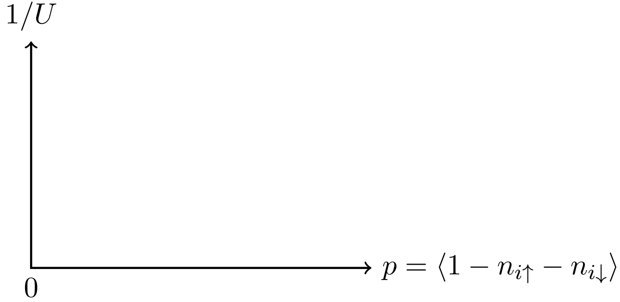


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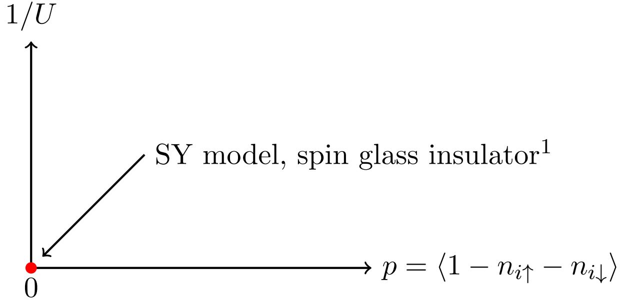


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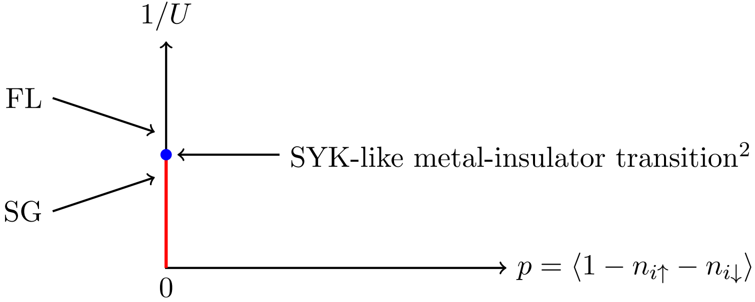


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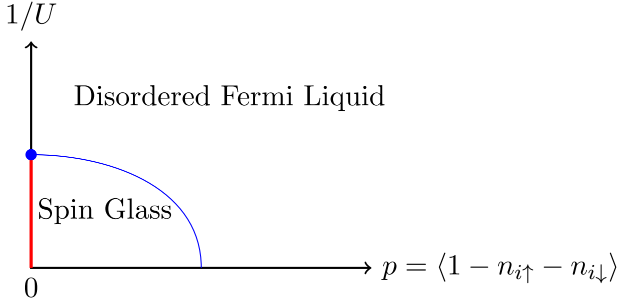


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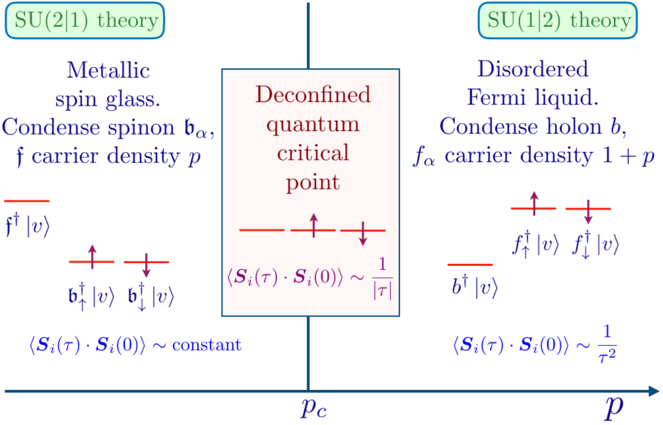
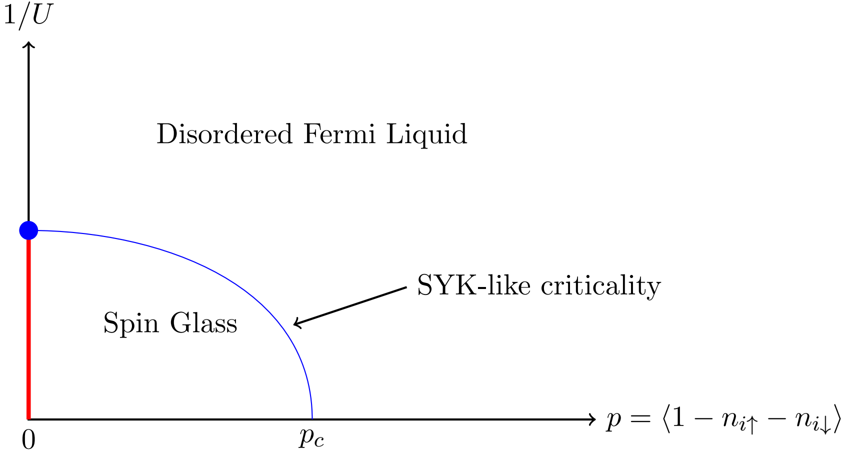



Zero-th order prediction of $p_c = 1/3$

Joshi et al. 2020.

# Hamiltonian and Phase Diagram



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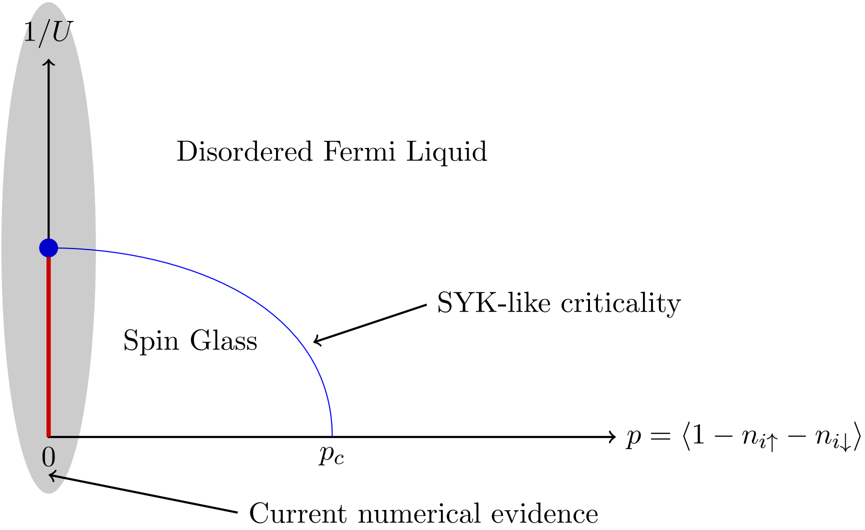

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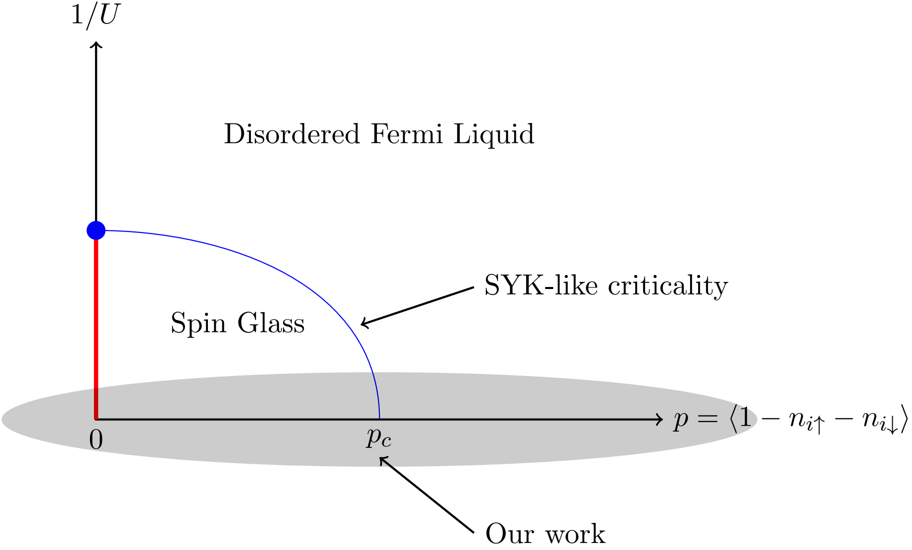

# Numerical Methods

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Wietek and Läuchli 2018.

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- Lanczos extends this to 18 sites, max dimension $\sim 8,000,000$
- Distributed memory parallelization allows for efficient usage of $\sim 100$ cores

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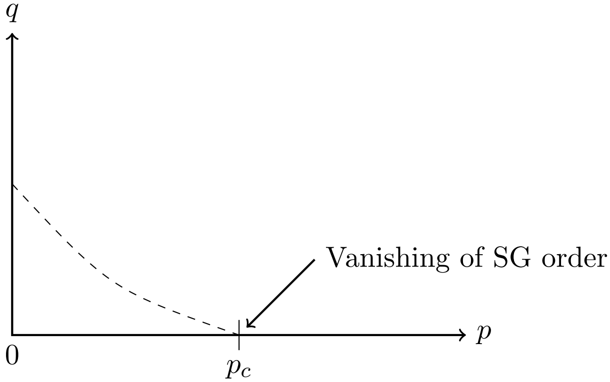
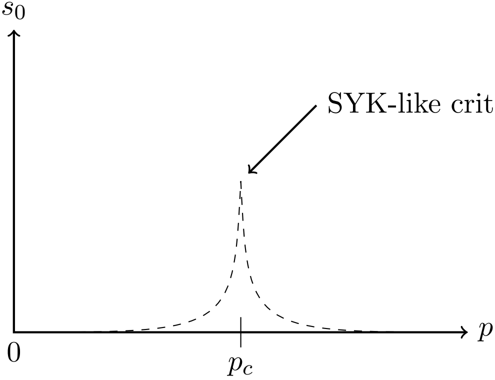
# Spin glass order measured by EA order parameter $q$

$$q = \lim_{t \to \infty} \langle \mathbf{S}_i(t) \cdot \mathbf{S}_i(0) \rangle \neq 0 \text{ for spin glass}$$



# SYK criticality measured by $T \to 0$ entropy density

$$s_0 = \lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} \neq 0 \text{ for SYK}$$



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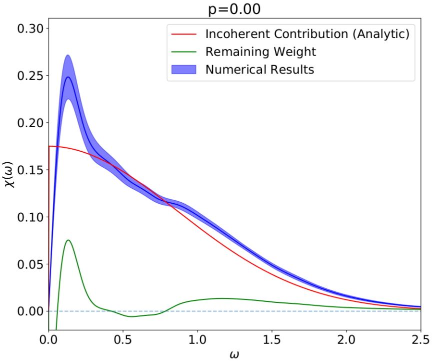
$\chi''(\omega) \sim \omega$ for FL

# $\chi''(\omega)$ at half-filling shows SG order




Arrachea and Rozenberg 2002.

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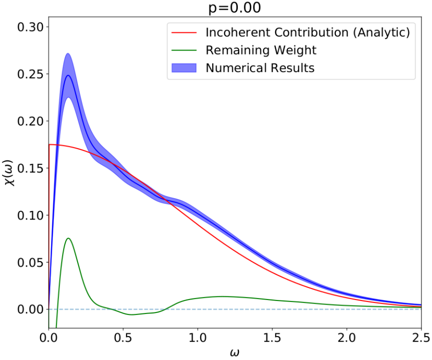
$$\chi''(\omega) = \underbrace{\chi''_{inc}(\omega)}_{N-\text{independent}}$$

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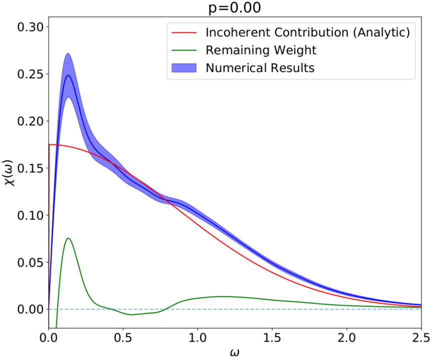
# $\chi''(\omega)$ at half-filling shows SG order

$$\chi''(\omega) = \underbrace{\chi''_{inc}(\omega)}_{N-\text{independent}} + \overbrace{\chi''_{low}(\omega) + \chi''_{high}(\omega)}^{\propto q}$$

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# $\chi''_{low}(\omega)$ asymptotes to a $\delta(\omega)$ at low frequency

$$\chi''_{low}(\omega) = A\omega \exp\left[-\frac{\omega^2}{2\Gamma^2}\right]$$

$\Gamma \to 0$ in the thermodynamic limit, whereas $\int_0^\infty \chi''_{low}(\omega)\, \mathrm{d}\omega \to q \neq 0$.

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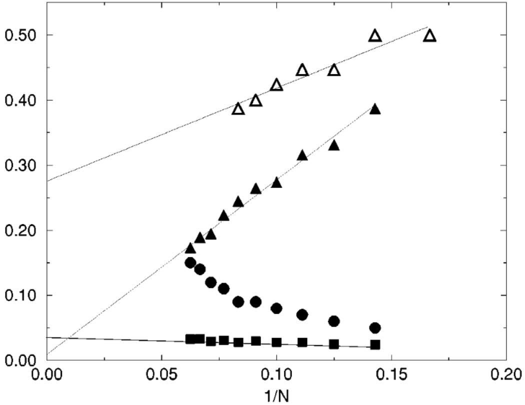


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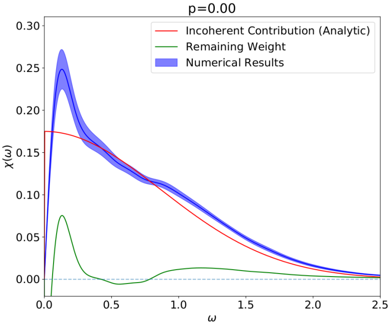
# $\chi''(\omega)$ for $p > 0$ has similar decomposition

$$\chi''(\omega) = \chi''_{inc}(\omega) + \chi''_{low}(\omega) + \chi''_{high}(\omega)$$

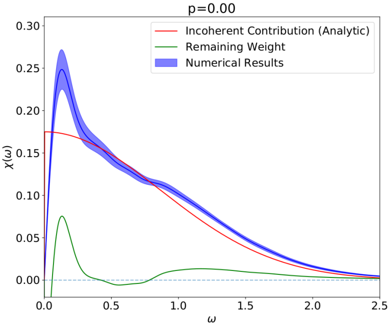
$$\chi''_{inc}(\omega) = C \exp\left[-\frac{\omega^2}{2J^2 S(S+1)}\right]$$



# $\chi''(\omega)$ for $p > 0$ has similar decomposition

$$\chi''(\omega) = \chi''_{inc}(\omega) + \chi''_{low}(\omega) + \chi''_{high}(\omega)$$

$$\chi''_{inc}(\omega) = nC \exp\left[-\frac{n\omega^2}{2J^2 S(S+1)}\right]$$



# Sum rule yields analytic prediction

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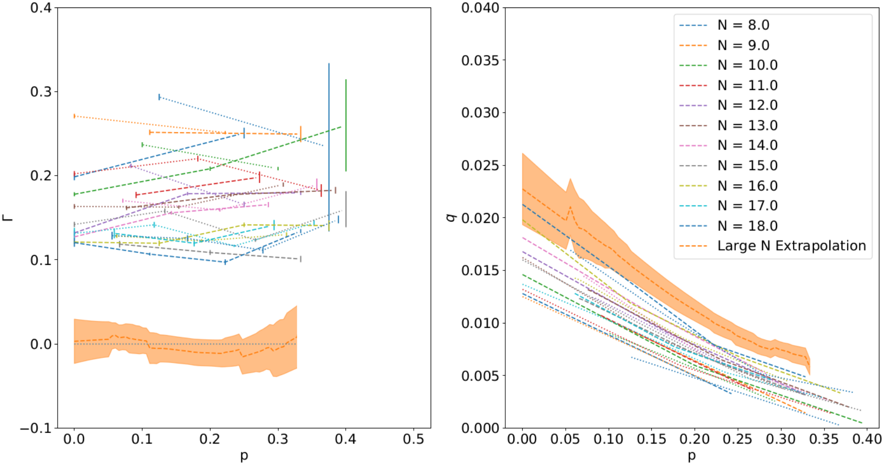
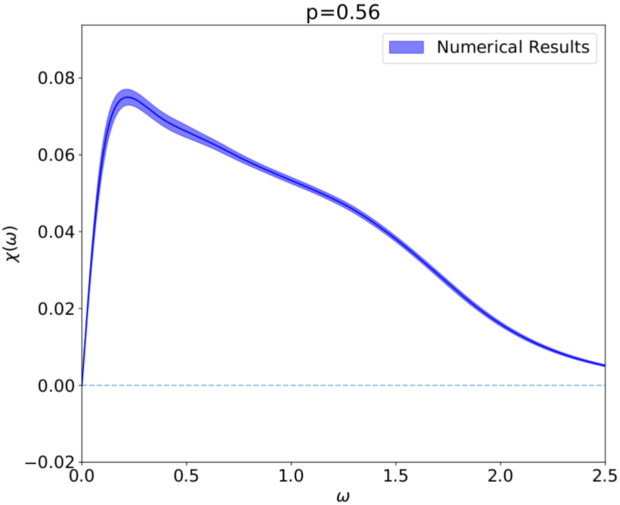
$$\int_0^\infty \mathrm{d}\omega\, \chi''(\omega) = \frac{n}{4}$$

At $q = 0$, $\chi''(\omega) = \chi''_{inc}(\omega)$, which gives $p_c = 0.423$.

# Large-$N$ extrapolation confirms stability of SG order



# Does vanishing of SG order correspond to onset of FL?



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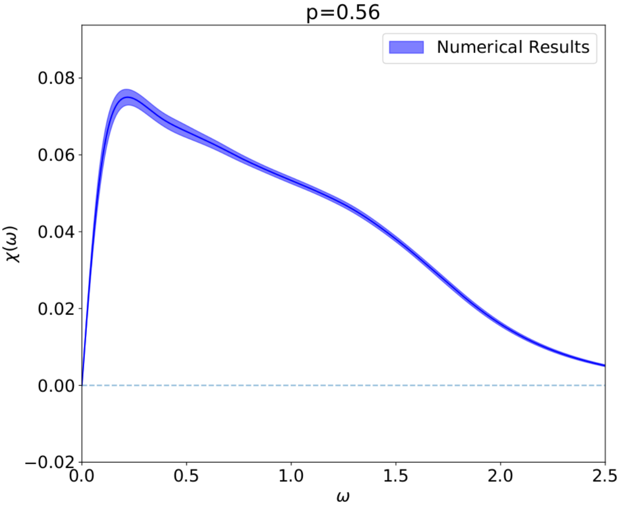


Consistent with $\chi''(\omega) \sim \omega$, $T > 0$ results should give a clearer answer

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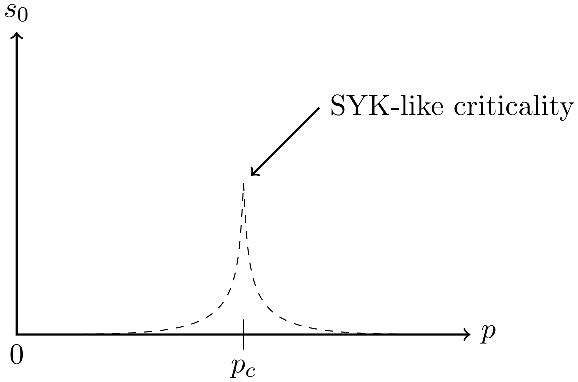
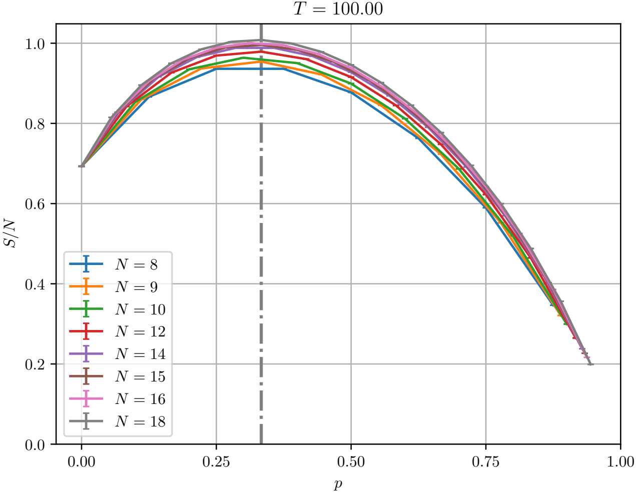
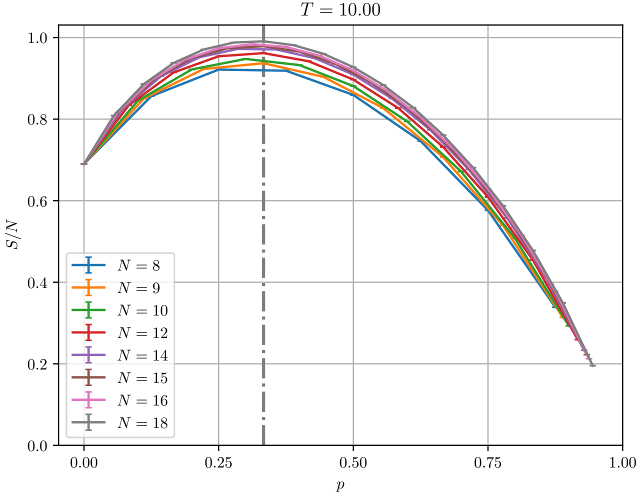


# Entropy at $T \gg 1$ determined by $\dim(\mathcal{H})$



# Maximum entropy shifts at lower temperature



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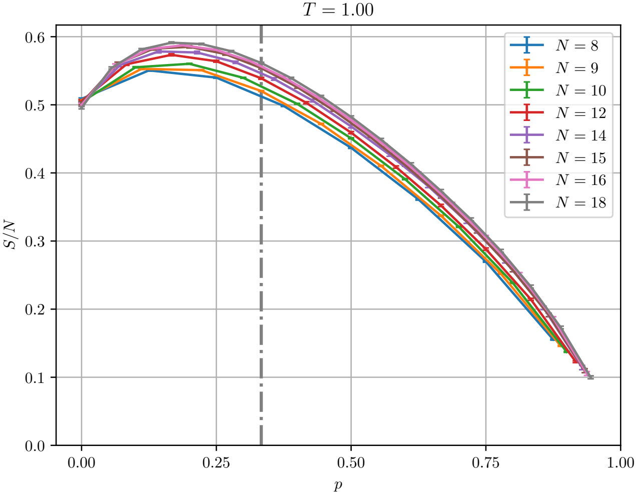

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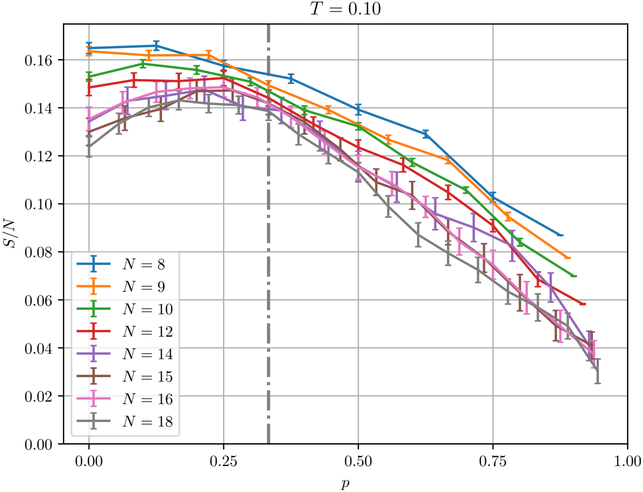
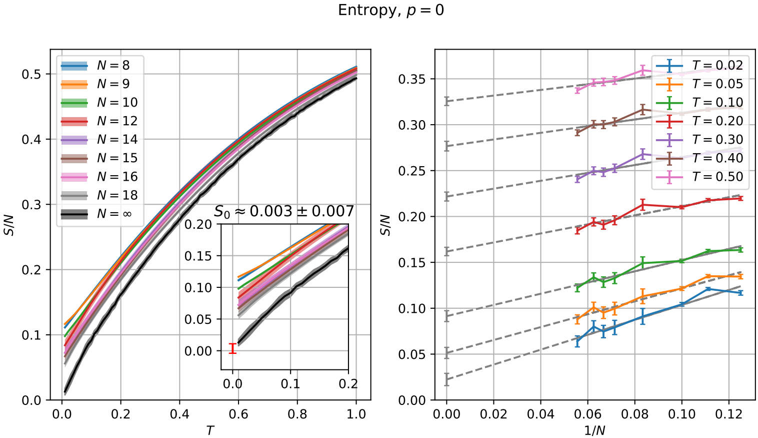


# Large-N extrapolation of entropy density



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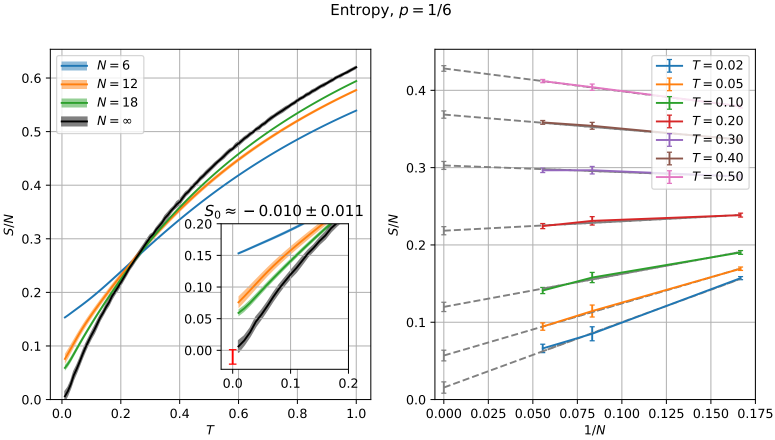

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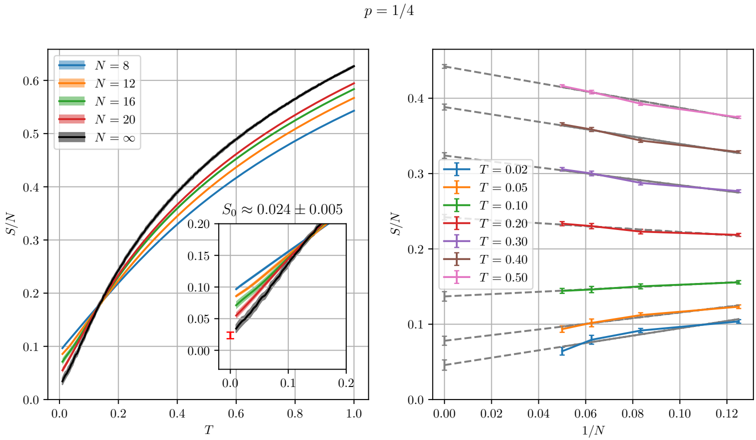

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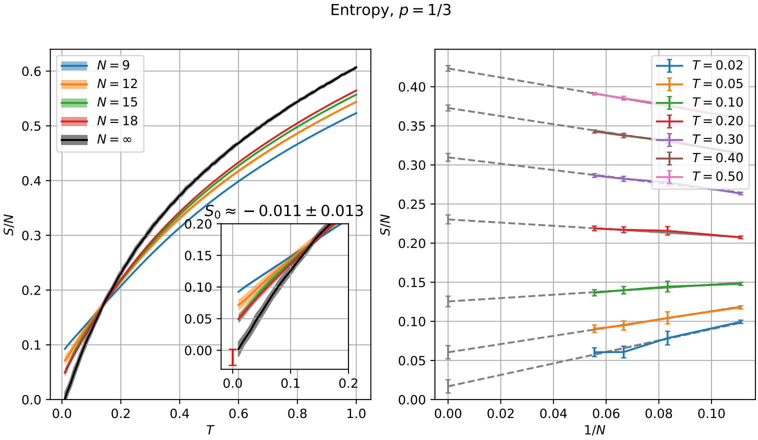

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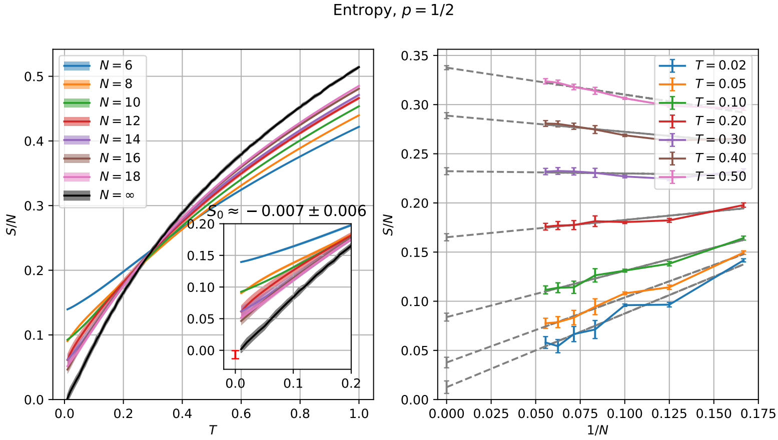
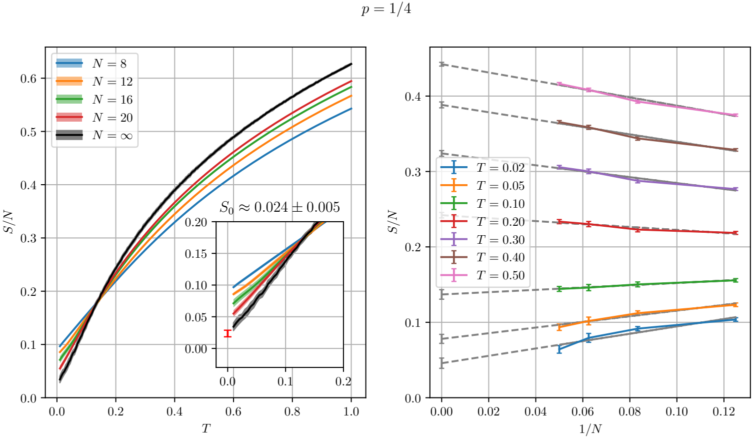


# Non-zero $s_0$ at $p = 1/4$ is extrapolation-dependent



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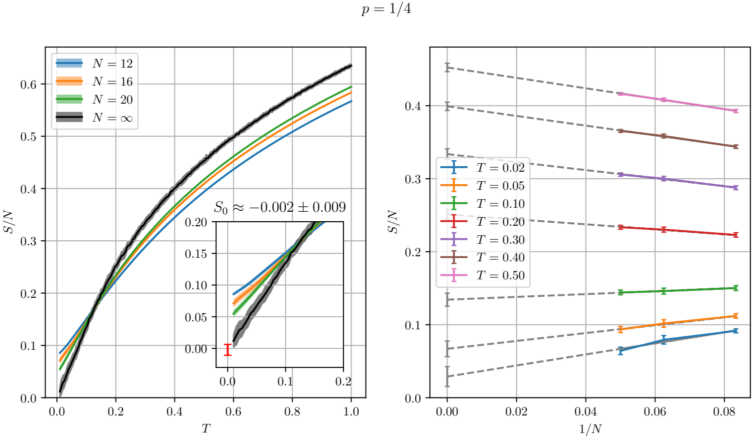

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- Separation could be due to finite-size effects
- More interestingly, two phase transitions?

# Future directions

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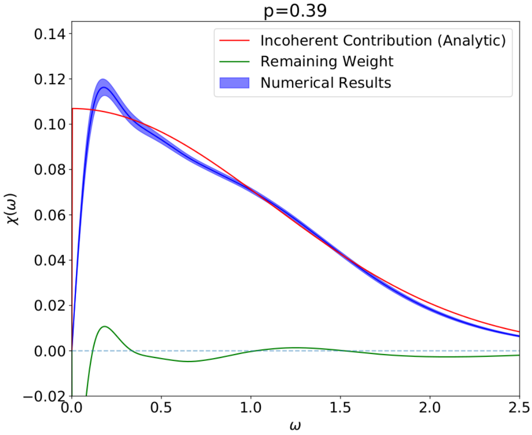
- Finite $U$ accessible with current code, more demanding due to larger Hilbert space
- Weaken the requirement of all-to-all interactions, sparse or power-law decay

# $\chi''(\omega)$ near criticality



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