Protection of parity-time symmetry in topological many-body systems: non-Hermitian toric code and fracton models

Henry Shackleton

Harvard University

July 1, 2020

Table of Contents

Table of Contents

Why Non-Hermiticity?

Why Non-Hermiticity?

Non-Hermiticity in condensed matter systems gives rise to phenomena that is unique to non-Hermitian systems and experimentally realizable.

Non-Hermiticity yields unique behavior

Kawabata et al., "Symmetry and Topology in Non-Hermitian Physics".

Non-Hermiticity yields unique behavior

- Enriched topological classification

Kawabata et al., "Symmetry and Topology in Non-Hermitian Physics".

Non-Hermiticity yields unique behavior

- Enriched topological classification
- Modified bulk-boundary correspondence

Kawabata et al., "Symmetry and Topology in Non-Hermitian Physics".

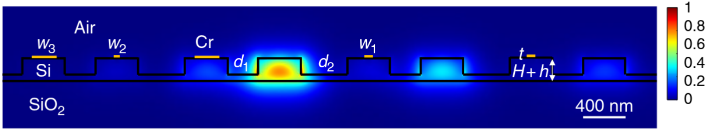
Non-Hermiticity yields unique behavior

- Enriched topological classification
- Modified bulk-boundary correspondence
- Exceptional points

Kawabata et al., "Symmetry and Topology in Non-Hermitian Physics".

Non-Hermiticity is experimentally realizable

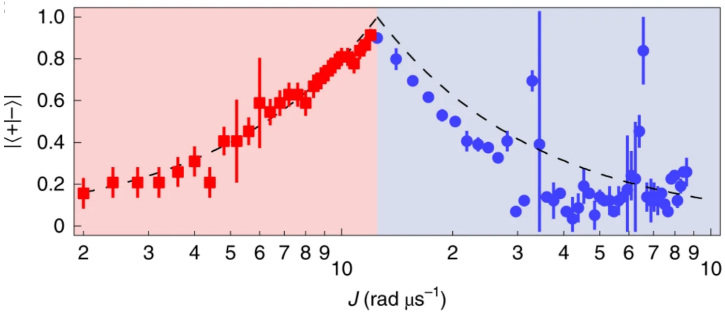
Photonic lattices with controlled gain and loss



Pan et al., “Photonic zero mode in a non-Hermitian photonic lattice”

Non-Hermiticity is experimentally realizable

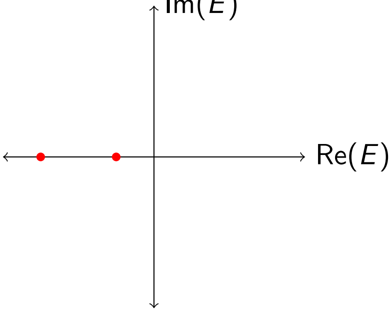
Open quantum systems (coherent dissipation from Lindblad equation)



Naghiloo et al., “Quantum state tomography across the exceptional point in a single dissipative qubit”

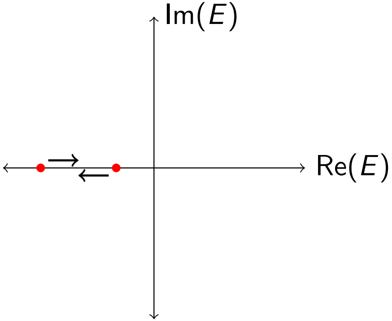
$\mathcal{PT}$ symmetry constrains eigenvalue spectra

- $[H, \mathcal{PT}] = 0$
- Eigenvalues come in complex conjugate pairs, $|\psi\rangle$ and $\mathcal{PT}\,|\psi\rangle$



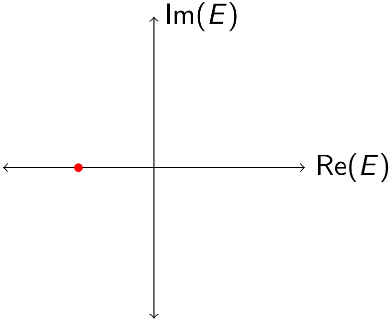
$\mathcal{PT}$ symmetry constrains eigenvalue spectra

- $[H, \mathcal{PT}] = 0$
- Eigenvalues come in complex conjugate pairs, $|\psi\rangle$ and $\mathcal{PT}\,|\psi\rangle$



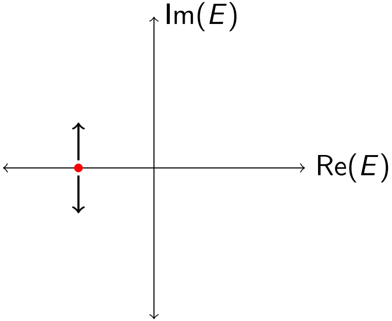
$\mathcal{PT}$ symmetry constrains eigenvalue spectra

- $[H, \mathcal{PT}] = 0$
- Eigenvalues come in complex conjugate pairs, $|\psi\rangle$ and $\mathcal{PT}\,|\psi\rangle$



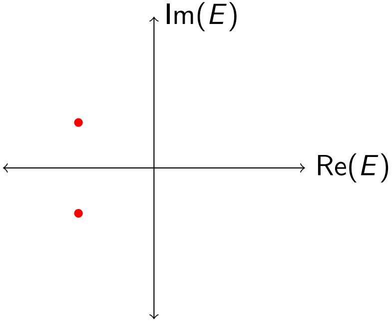
$\mathcal{PT}$ symmetry constrains eigenvalue spectra

- $[H, \mathcal{PT}] = 0$
- Eigenvalues come in complex conjugate pairs, $|\psi\rangle$ and $\mathcal{PT}\,|\psi\rangle$



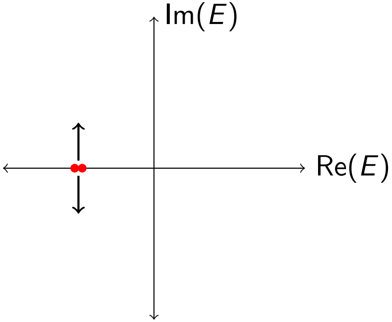
$\mathcal{PT}$ symmetry constrains eigenvalue spectra

- $[H, \mathcal{PT}] = 0$
- Eigenvalues come in complex conjugate pairs, $|\psi\rangle$ and $\mathcal{PT}\,|\psi\rangle$



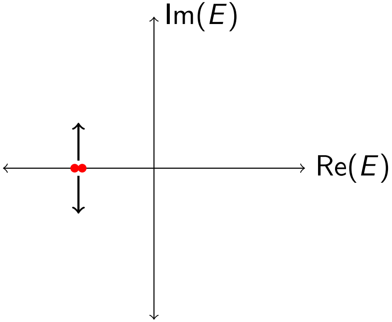
Degeneracies in $\mathcal{PT}$-symmetric Hamiltonians

Degenerate or nearly-degenerate eigenvalues are not afforded the same protection against complexity.



Degeneracies in $\mathcal{PT}$-symmetric Hamiltonians

Degenerate or nearly-degenerate eigenvalues are not afforded the same protection against complexity.



How do topological degeneracies behave in the presence of $\mathcal{PT}$-symmetric perturbations?

Pseudo-Hermiticity generalizes $\mathcal{PT}$ symmetry

$$\eta^{-1} H \eta = H^\dagger$$

Pseudo-Hermiticity generalizes $\mathcal{PT}$ symmetry

$$\eta^{-1} H \eta = H^{\dagger}$$

$$H = \underbrace{H_0}_{[H_0,\eta]=0} + \underbrace{iV}_{\{V,\eta\}=0}$$

Degenerate behavior constrained by η

- If the unperturbed degenerate eigenstates have the same eigenvalue under η, they will stay real

Krein, "A generalization of some investigations of linear differential equations with periodic coefficients".

Degenerate behavior constrained by η

- If the unperturbed degenerate eigenstates have the same eigenvalue under η, they will stay real
- $\eta^{-1} H \eta = H^\dagger$ implies $H = H^\dagger$ in degenerate subspace

Krein, "A generalization of some investigations of linear differential equations with periodic coefficients".

Degenerate behavior constrained by η

- If the unperturbed degenerate eigenstates have the same eigenvalue under η, they will stay real
- $\eta^{-1} H \eta = H^\dagger$ implies $H = H^\dagger$ in degenerate subspace
- H_{eff} stays Hermitian at all orders in perturbation theory

Krein, "A generalization of some investigations of linear differential equations with periodic coefficients".

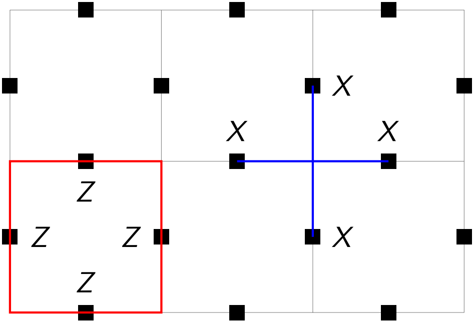
Degenerate behavior constrained by η

- If the unperturbed degenerate eigenstates have the same eigenvalue under η, they will stay real
- $\eta^{-1} H \eta = H^\dagger$ implies $H = H^\dagger$ in degenerate subspace
- H_{eff} stays Hermitian at all orders in perturbation theory
- Eigenvalues will *generally* become complex when η is non-trivial in degenerate subspace

Krein, "A generalization of some investigations of linear differential equations with periodic coefficients".

Table of Contents

Topological Order in Toric Code



$$H = -\sum_c {\color{blue}A_c} - \sum_p {\color{red}B_p}$$

Kitaev, "Fault-tolerant quantum computation by anyons".

Topological Order in Toric Code

- For ground states, $A_c = B_p = +1$

Topological Order in Toric Code

- For ground states, $A_c = B_p = +1$
- On a torus, non-contractible loop operators lead to a fourfold GSD

Unique choice of symmetry gives generic pseudo-Hermitian perturbations

Pseudo-Hermiticity requires $\{\eta, V\} = 0$.

Unique choice of symmetry gives generic pseudo-Hermitian perturbations

Pseudo-Hermiticity requires $\{\eta, V\} = 0$. If we assume:

- V can be disordered

Unique choice of symmetry gives generic pseudo-Hermitian perturbations

Pseudo-Hermiticity requires $\{\eta, V\} = 0$. If we assume:

- V can be disordered
- V can have support on the entire lattice

Unique choice of symmetry gives generic pseudo-Hermitian perturbations

Pseudo-Hermiticity requires $\{\eta, V\} = 0$. If we assume:

- V can be disordered
- V can have support on the entire lattice

then we have a unique choice.

The TC symmetries which admits generic pseudo-Hermitian perturbations are $\eta = \prod_i X_i \,, \prod_i Y_i \,, \prod_i Z_i$

What perturbations are permitted by η?

What perturbations are permitted by η?

Choosing $\eta = \prod_i X_i$ for concreteness, we can have

What perturbations are permitted by η?

Choosing $\eta = \prod_i X_i$ for concreteness, we can have

- $i \sum_i Z_i$

What perturbations are permitted by η?

Choosing $\eta = \prod_i X_i$ for concreteness, we can have

- $i \sum_i Z_i$
- $i \sum_i g_{x,i} Y_i + g_{z,i} Z_i$

What perturbations are permitted by η?

Choosing $\eta = \prod_i X_i$ for concreteness, we can have

- $i \sum_i Z_i$
- $i \sum_i g_{x,i} Y_i + g_{z,i} Z_i$
- $i \sum_{i<j<k} g_{ijk} Z_i Z_j Z_k$

What perturbations are permitted by η?

Choosing $\eta = \prod_i X_i$ for concreteness, we can have

- $i \sum_i Z_i$
- $i \sum_i g_{x,i} Y_i + g_{z,i} Z_i$
- $i \sum_{i<j<k} g_{ijk} Z_i Z_j Z_k$
- Essentially *any perturbation* with an odd number of Y and Z operators

What perturbations are permitted by η?

Choosing $\eta = \prod_i X_i$ for concreteness, we can have

- $i \sum_i Z_i$
- $i \sum_i g_{x,i} Y_i + g_{z,i} Z_i$
- $i \sum_{i<j<k} g_{ijk} Z_i Z_j Z_k$
- Essentially *any perturbation* with an odd number of Y and Z operators

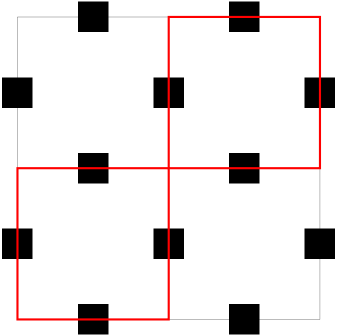
Hermitian perturbations are also allowed, provided they commute with η

Eigenvalues of η are sensitive to system size

Eigenvalues of η are sensitive to system size

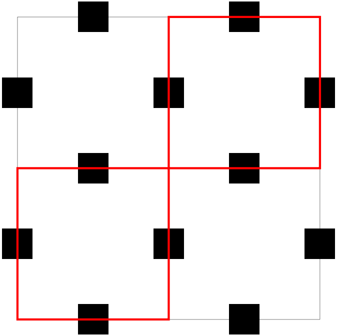
On an even-by-even lattice, η is the product of stabilizers.

$$\prod_i Z_i =$$



Eigenvalues of η are sensitive to system size

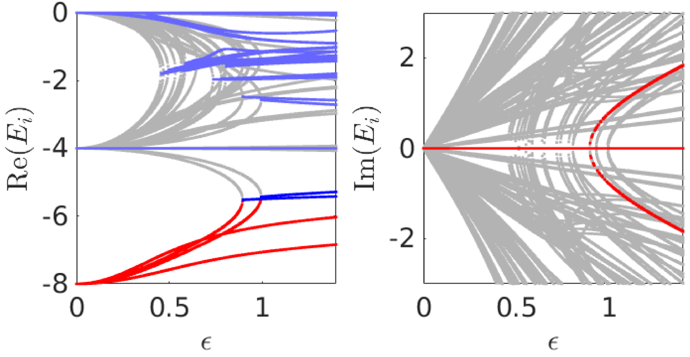
On an even-by-even lattice, η is the product of stabilizers.

$\prod_i Z_i =$ 

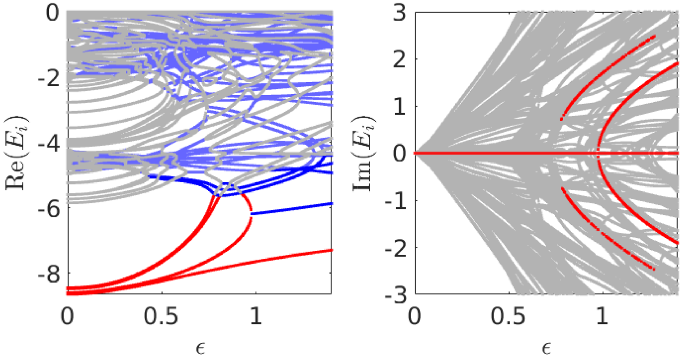
For any other system size, η is the product of stabilizers and logical string operators.

Pseudo-Hermitian perturbations sensitive to system size

For generic pseudo-Hermitian/$\mathcal{PT}$-symmetric perturbations, the ground state subspace of the toric code stays real *if the system size is even in all directions.*

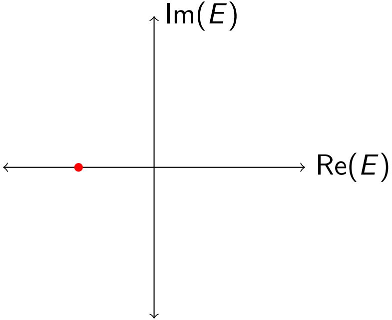


$\mathcal{PT}$-symmetry protection does not require fine-tuning



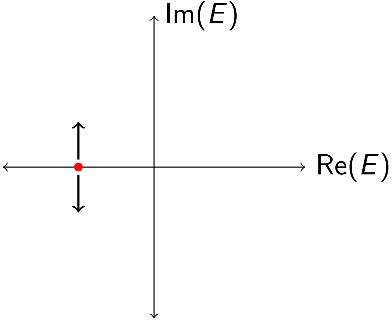
Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise



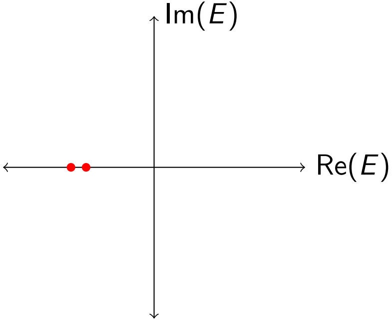
Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise



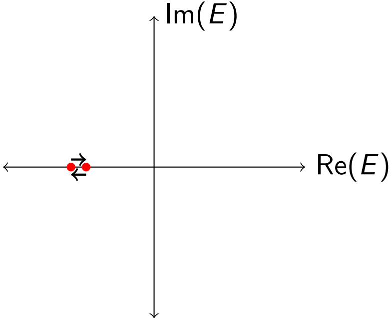
Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise



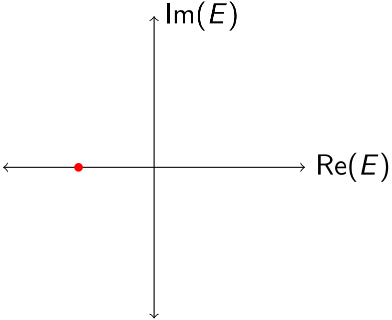
Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise



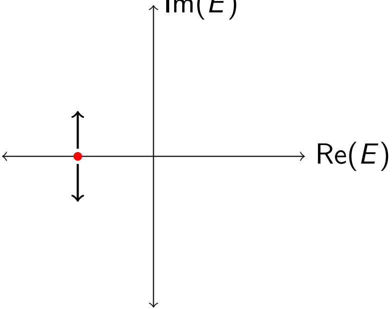
Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise



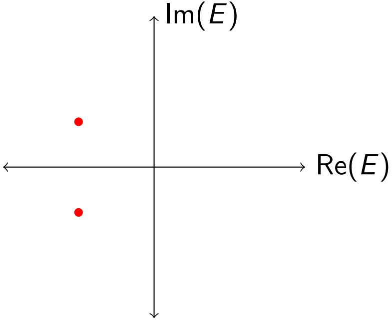
Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise



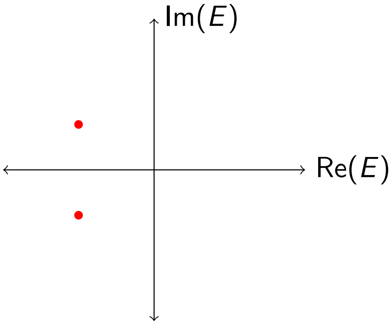
Pseudo-Hermitian perturbations sensitive to system size

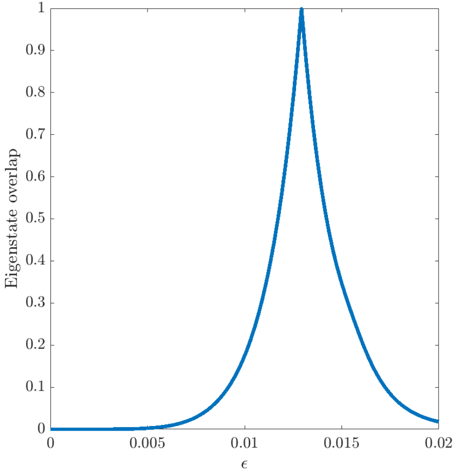
If the system size is not even-by-even, exceptional points generically arise



Pseudo-Hermitian perturbations sensitive to system size

If the system size is not even-by-even, exceptional points generically arise





$\mathcal{PT}$-symmetry in the Toric Code

$\mathcal{PT}$-symmetry in the Toric Code

- Even-by-even lattice $\Rightarrow$ $\mathcal{PT}$-symmetry is preserved, effective Hamiltonian is Hermitian

$\mathcal{PT}$-symmetry in the Toric Code

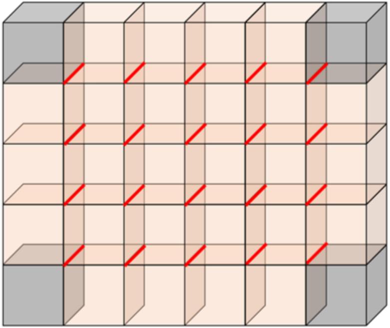
- Even-by-even lattice $\Rightarrow$ $\mathcal{PT}$-symmetry is preserved, effective Hamiltonian is Hermitian
- Not even-by-even lattice $\Rightarrow$ $\mathcal{PT}$-symmetry generically broken, can create exceptional points

$\mathcal{PT}$-symmetry in the Toric Code

- Even-by-even lattice $\Rightarrow$ $\mathcal{PT}$-symmetry is preserved, effective Hamiltonian is Hermitian
- Not even-by-even lattice $\Rightarrow$ $\mathcal{PT}$-symmetry generically broken, can create exceptional points
- Geometric criterion related to covering of a lattice by stabilizers

Table of Contents

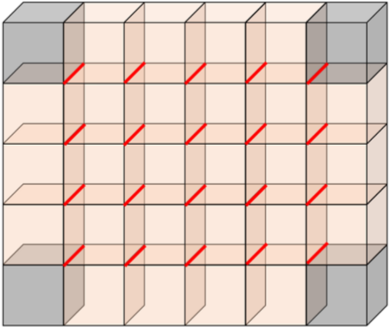
Overview of fracton models



- Excitations with restricted mobility

Pretko, Chen, and You, "Fracton phases of matter".

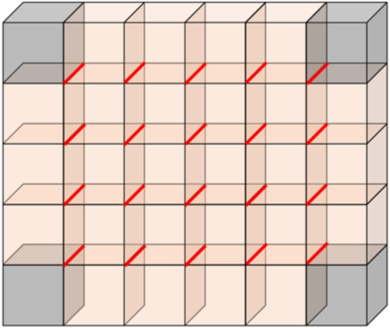
Overview of fracton models



- Excitations with restricted mobility
- GSD scales exponentially in system size

Pretko, Chen, and You, "Fracton phases of matter".

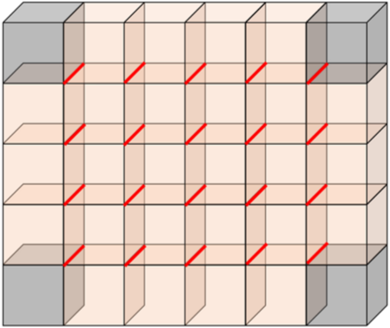
Overview of fracton models



- Excitations with restricted mobility
- GSD scales exponentially in system size
- GSD not connected to any individual symmetry, "topological"

Pretko, Chen, and You, "Fracton phases of matter".

Overview of fracton models



- Excitations with restricted mobility
- GSD scales exponentially in system size
- GSD not connected to any individual symmetry, "topological"
- Does the GSD have a similar $\mathcal{PT}$-symmetry breaking protection?

Pretko, Chen, and You, "Fracton phases of matter".

Pseudo-Hermiticity approach generalizes from toric code

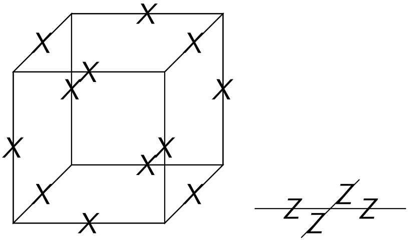
Pseudo-Hermiticity approach generalizes from toric code

- $\eta = \prod_i X_i \,, \prod_i Y_i \,, \prod_i Z_i$

Pseudo-Hermiticity approach generalizes from toric code

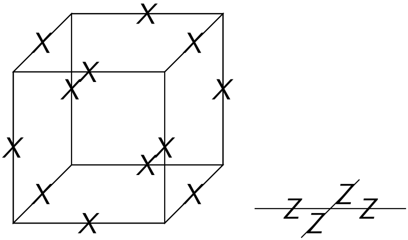
- $\eta = \prod_i X_i \,, \prod_i Y_i \,, \prod_i Z_i$
- Protection of $\mathcal{PT}$ symmetry $\Leftrightarrow$ non-overlapping covering of stabilizers

Topological Order in the X-Cube model



Vijay, Haah, and Fu, "Fracton topological order, generalized lattice gauge theory, and duality".

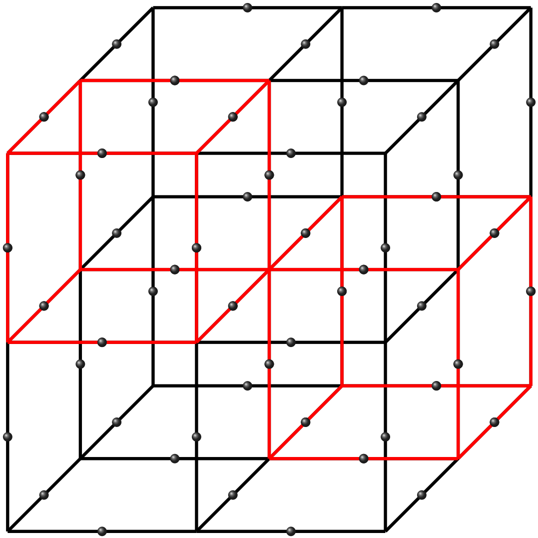
Topological Order in the X-Cube model



$$\log_2(\mathrm{GSD}) = 2L_x + 2L_y + 2L_z - 3$$

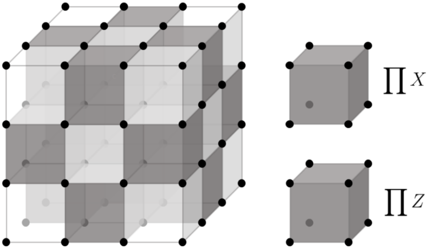
Vijay, Haah, and Fu, "Fracton topological order, generalized lattice gauge theory, and duality".

Even/odd classification is the same as toric code

$\prod_i X_i =$ 

Other fracton models with $\mathcal{PT}$-symmetry protection

Checkerboard model:

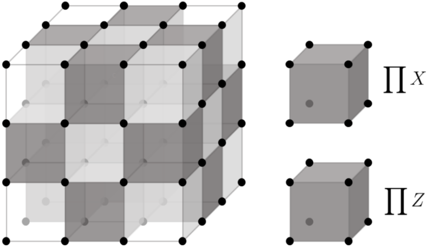


Vijay, Haah, and Fu, "Fracton topological order, generalized lattice gauge theory, and duality".

Shirley, Slagle, and Chen, "Foliated fracton order in the checkerboard model".

Other fracton models with $\mathcal{PT}$-symmetry protection

Checkerboard model:



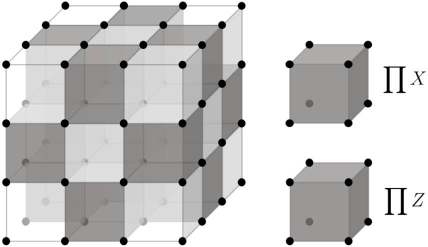
Covering *always* exists

Vijay, Haah, and Fu, "Fracton topological order, generalized lattice gauge theory, and duality".

Shirley, Slagle, and Chen, "Foliated fracton order in the checkerboard model".

Other fracton models with $\mathcal{PT}$-symmetry protection

Checkerboard model:



Covering *always* exists

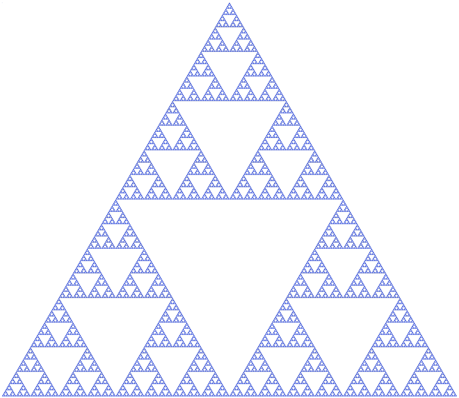
Also generalizes to Majorana version, $\eta = \prod_i \gamma_i$

Vijay, Haah, and Fu, "Fracton topological order, generalized lattice gauge theory, and duality".

Shirley, Slagle, and Chen, "Foliated fracton order in the checkerboard model".

Other fracton models with $\mathcal{PT}$-symmetry protection

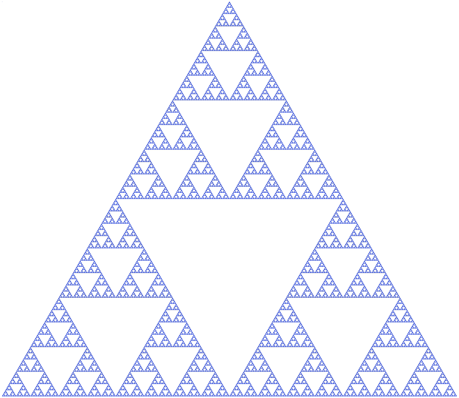
Quantum fractal spin liquids:



Yoshida, "Exotic topological order in fractal spin liquids".

Other fracton models with $\mathcal{PT}$-symmetry protection

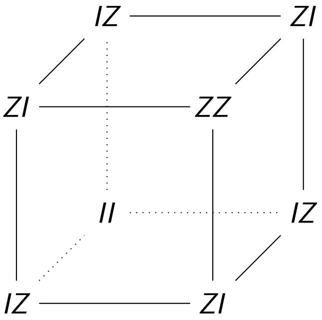
Quantum fractal spin liquids:



η *always* commutes with logical string operators

Yoshida, "Exotic topological order in fractal spin liquids".

Topological order in Haah's cubic codes



- 17 stabilizer codes
- Two qubits per site
- Existence of a covering can be checked by algebraic techniques

Haah, "Local stabilizer codes in three dimensions without string logical operators".

Existence of a stabilizer covering depends on system size

System Size	CC_1	CC_2 CC_3 CC_6 CC_9	CC_4	CC_5 CC_8 CC_{10} CC_{16}	CC_7	CC_{11} CC_{12} CC_{14} CC_{15}	CC_{13}	CC_{17}
E $\times$ E $\times$ E E $\times$ E $\times$ e	✓	✓	✓	✓	✓	✓	✓	✓
e $\times$ E $\times$ E o $\times$ E $\times$ E E $\times$ e $\times$ E E $\times$ e $\times$ e	✓	✓	✓	✓	✓	✓	✓	✗
e $\times$ e $\times$ E e $\times$ E $\times$ E E $\times$ o $\times$ E e $\times$ e $\times$ e	✓	✓	✓	✓	✓	✓	✗	✗
E $\times$ E $\times$ o	✓	✓	✓	✓	✓	✗	✗	✓
e $\times$ o $\times$ E E $\times$ o $\times$ e e $\times$ o $\times$ e	✗	✓	✓	✓	✓	✓	✗	✗
e $\times$ E $\times$ o E $\times$ e $\times$ o e $\times$ e $\times$ o e $\times$ o $\times$ o E $\times$ o $\times$ o	✗	✓	✗	✓	✓	✗	✗	✗
o $\times$ e $\times$ E o $\times$ E $\times$ e o $\times$ e $\times$ e	✗	✗	✓	✓	✓	✓	✗	✗
o $\times$ o $\times$ E o $\times$ o $\times$ e	✗	✗	✗	✗	✓	✓	✗	✗
o $\times$ E $\times$ o o $\times$ e $\times$ o o $\times$ o $\times$ o	✗	✗	✗	✗	✗	✗	✗	✗

Dua et al., "Bifurcating entanglement-renormalization group flows of fracton stabilizer models".

Existence of a stabilizer covering depends on system size

System Size	CC_1	CC CC_3 CC_6 CC_9	CC_4	CC_5 CC_8 CC_{10} CC_{16}	CC_7	CC_{11} CC_{12} CC_{14} CC_{15}	CC_{13}	CC_{17}
E × E × E E × E × e	✓	✓	✓	✓	✓	✓	✓	✓
e × E × E o × E × E E × e × E E × e × e	✓	✓	✓	✓	✓	✓	✓	✗
e × e × E e × E × E E × o × E e × e × e	✓	✓	✓	✓	✓	✓	✗	✗
E × E × o	✓	✓	✓	✓	✓	✗	✗	✓
e × o × E E × o × e e × o × e	✗	✓	✓	✓	✓	✓	✗	✗
e × E × o E × e × o e × e × o e × o × o E × o × o	✗	✓	✗	✓	✓	✗	✗	✗
o × e × E o × E × e o × e × e	✗	✗	✓	✓	✓	✓	✗	✗
o × o × E o × o × e	✗	✗	✗	✗	✓	✓	✗	✗
o × E × o o × e × o o × o × o	✗	✗	✗	✗	✗	✗	✗	✗

Dua et al., "Bifurcating entanglement-renormalization group flows of fracton stabilizer models".

Table of Contents

Conclusions

Conclusions

- $\mathcal{PT}$ symmetry/pseudo-Hermiticity admits controlled non-Hermitian effects

Conclusions

- $\mathcal{PT}$ symmetry/pseudo-Hermiticity admits controlled non-Hermitian effects
- Degeneracies/near-degeneracies can spontaneously break $\mathcal{PT}$-symmetry

Conclusions

- $\mathcal{PT}$ symmetry/pseudo-Hermiticity admits controlled non-Hermitian effects
- Degeneracies/near-degeneracies can spontaneously break $\mathcal{PT}$-symmetry
- Topological degeneracies can give $\mathcal{PT}$-symmetry protection against generic perturbations

Conclusions

- $\mathcal{PT}$ symmetry/pseudo-Hermiticity admits controlled non-Hermitian effects
- Degeneracies/near-degeneracies can spontaneously break $\mathcal{PT}$-symmetry
- Topological degeneracies can give $\mathcal{PT}$-symmetry protection against generic perturbations
- $\mathcal{PT}$-symmetry protection related to stabilizer coverings of lattice

Open Questions

Open Questions

- Interplay between exceptional points and topological order

Open Questions

- Interplay between exceptional points and topological order
- Hermitian applications?

Open Questions

- Interplay between exceptional points and topological order
- Hermitian applications?
- Novel way of classifying fracton phases