Exactly solvable dissipative spin liquids

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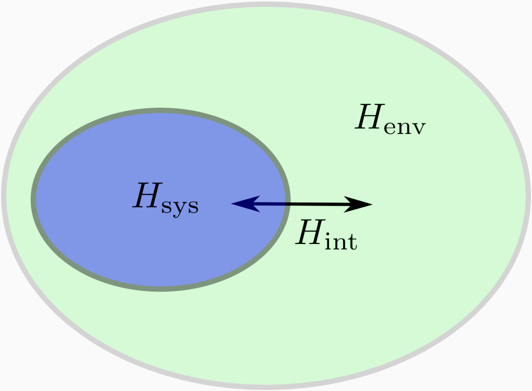
Open quantum systems = non-equilibrium phases?



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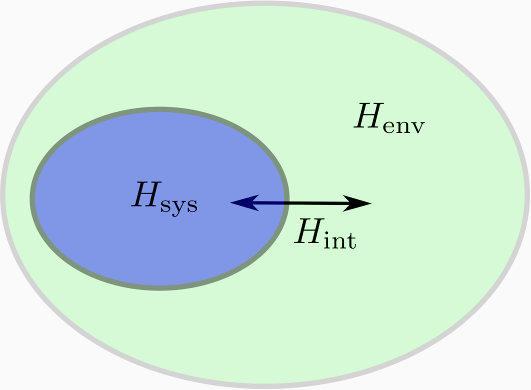

Simplest model: $\rho_{\text{sys}} = e^{-\beta H_{\text{sys}}}$

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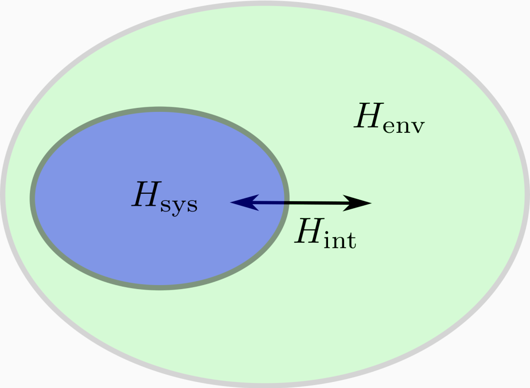

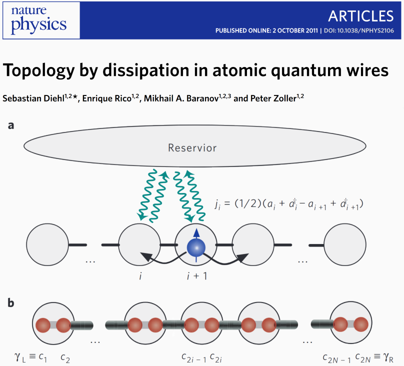

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Emerging ideas: engineering of environmental couplings lead to richer physics

Generic open quantum system dynamics described by Lindbladian

Lindblad equation - general Markovian evolution satisfying CPTP conditions

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathcal{L}[\rho] = -i[H, \rho] + \sum_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\left\{ L_i^\dagger L_i, \rho \right\} \right)$$

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Treat $\mathcal{L}$ as Hamiltonian acting on "doubled" Hilbert space of operators,

$$\mathcal{O} = \sum_{ij} \mathcal{O}_{ij} \left|\psi_i\right\rangle \left\langle\psi_j\right| \Rightarrow \sum_{ij} \mathcal{O}_{ij} \left|\psi_i\right\rangle \otimes \left|\psi_j\right\rangle$$

and $\langle \mathcal{O}_1 | \mathcal{O}_2 \rangle \sim \mathrm{Tr}\, \mathcal{O}_1^\dagger \mathcal{O}_2$.

$$i\mathcal{L} = H_{\text{eff}} \otimes \mathbb{1} - \mathbb{1} \otimes H_{\text{eff}}^\dagger + i\gamma \sum_i L_i \otimes L_i^\dagger$$

$$H_{\text{eff}} \equiv H - \frac{i\gamma}{2} \sum_i L_i^\dagger L_i$$

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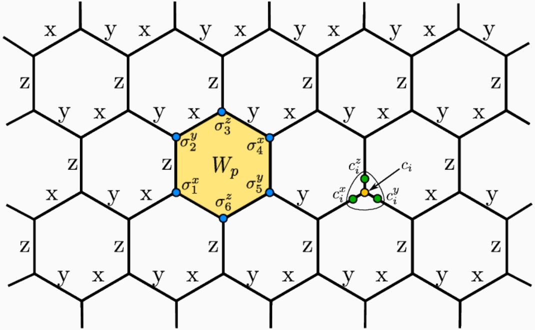
Part 1: Define our model and demonstrate exact solvability

Part 2: Interpret meaning of ground states, excited states, etc

Exact solvability in the Kitaev honeycomb model: a review

$$H = J_x \sum_{\text{x-links}} \sigma_j^x \sigma_k^x + J_y \sum_{\text{y-links}} \sigma_j^y \sigma_k^y$$
$$+ J_z \sum_{\text{z-links}} \sigma_j^z \sigma_k^z$$

$$\sigma_j^\alpha = i c_j b_j^\alpha \quad c_j \prod_\alpha b_j^\alpha = 1$$



Kitaev 2006; Wu, Arovas, and Hung 2009.

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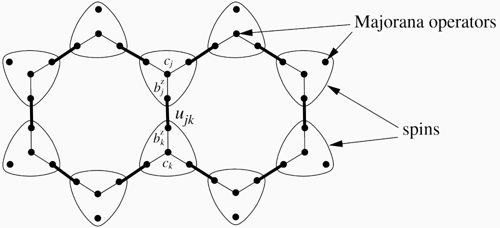

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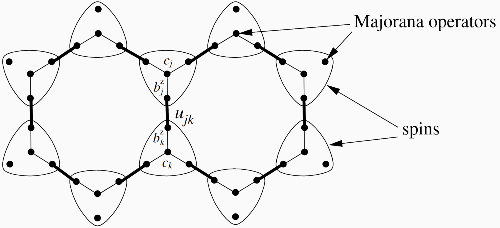


- General construction - n anti-commuting matrices on a lattice of coordination number n
- Accomplished by Γ matrices, $\left\{\Gamma^a, \Gamma^b\right\} = 2\delta_{ab}$

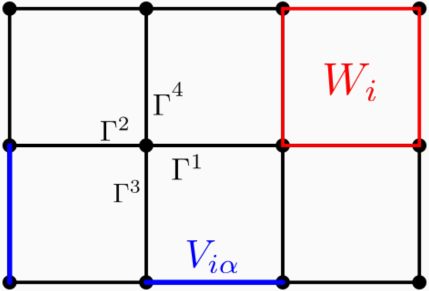
Kitaev 2006; Wu, Arovas, and Hung 2009.

Exactly solvable bilayer models lift to Lindbladian picture

Define Kitaev-like model on bilayer square lattice, coordination number of 5 requires Γ^a, $a = 1, \ldots, 5$ (microscopic DOFs are spin-3/2 or spin/orbital)

$$H = \sum_j \left[J_x \Gamma_j^1 \Gamma_{j+\widehat{x}}^2 + J_y \Gamma_j^3 \Gamma_{j+\widehat{y}}^4 \right]$$

Conserved quantities: flux operators W_i, bond operators $V_{i\alpha}$



Yao, Zhang, and Kivelson 2009.

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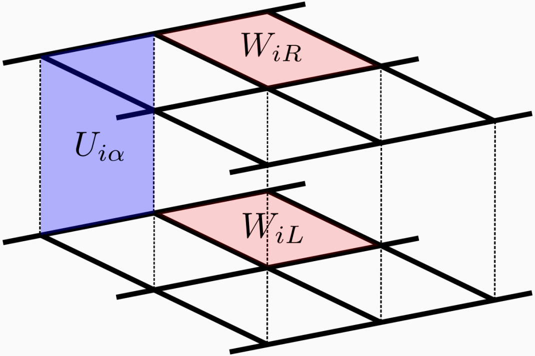
Add quantum jump operator $L_j = \Gamma^5_j$, representation in doubled Hilbert space is

$$i\mathcal{L} = H[\Gamma_R] - H[\Gamma_L] + i\gamma \sum_j \Gamma^5_{jL}\Gamma^5_{jR} - i\gamma N$$

Conserved fluxes W_{iR}, W_{iL} along with conserved *superoperators*

$$U_{i\alpha}[\rho] = V_{i\alpha}\rho V_{i\alpha}$$



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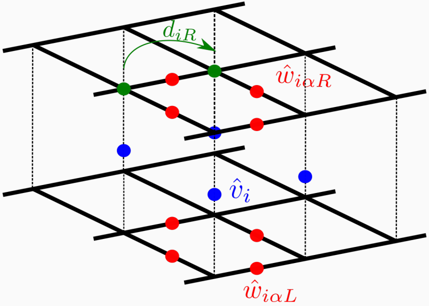
Exact solvability through Majorana decomposition

Use mapping $\Gamma^a_{jR} = i d_{jR} c^a_{jR}$ and likewise for Γ^a_{jL}

$$i\mathcal{L} = \sum_{\ell=L,R} \sum_i s_\ell \left[J_x \, \widehat{w}_{i,x,\ell} \, i d_{i,\ell} d_{i+\widehat{x},\ell} + J_y \, \widehat{w}_{i,y,\ell} \, i d_{i,\ell} d_{i+\widehat{y},\ell} \right] - \gamma \sum_i \widehat{v}_i \, d_{i,R} d_{i,L} - i\gamma N$$

$s_R = 1 \, , s_L = -1$

$c^a_{j\ell}$ bilinears, conserved quantities ± 1



Lindblad equation can be expressed as Majorana fermions coupled to static $\mathbb{Z}_2$ gauge fields - free fermions in each gauge sector.

What is the physical interpretation of these solutions?

For $t \to \infty$, initial states asymptote to steady-state solutions ρ_{ss}, where $\mathcal{L} |\rho_{ss}\rangle = 0$

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Symmetry-based characterization

$\langle A | \rho \rangle = 0$ unless A and ρ are in the same symmetry sector. Extensive number of flux operators makes this a powerful tool!

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Excitations define *Liouvillian gap* λ, transient behavior over timescales $t \sim \frac{1}{\lambda}$

Particle-based characterization

Different "particles" correspond to equilibration of different observables

$$|\rho_{ss}\rangle + |a\rangle \xrightarrow[t \sim \lambda_a^{-1}]{} |\rho_{ss}\rangle$$

What are the steady-state solutions?

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Lieb's theorem analogy

Eigenfunctions with non-zero trace must be steady-state solutions by trace-preservation of the Lindbladian. $\langle \mathbb{1} | \mathcal{O} \rangle \neq 0$ means that $|\mathbb{1}\rangle$ and $|\mathcal{O}\rangle$ must be in the same symmetry sector

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Interlayer flux constraint

$$U_{i\alpha}[\mathbb{1}] = V_{i\alpha} \mathbb{1} V_{i\alpha} = \mathbb{1}$$

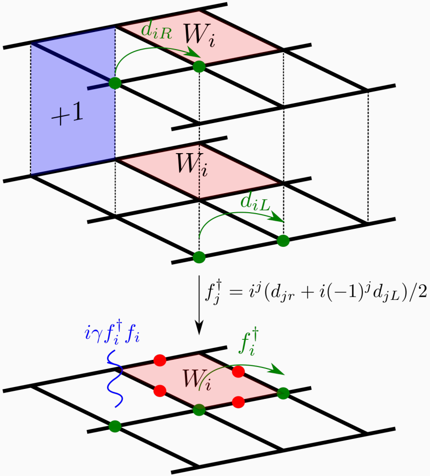
Enforces $U_{i\alpha} = 1$

Intralayer flux constraint

$$W_{iR}[\mathbb{1}] = W_{iL}[\mathbb{1}]$$

Enforces $W_{iR} = W_{iL} \equiv W_i$

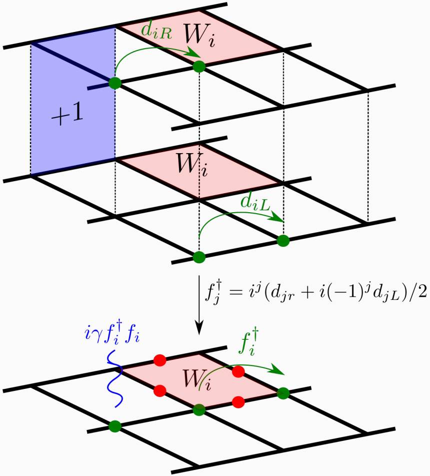
Complex fermion representation reveals steady-state solutions



$$i\mathcal{L} = J \sum_j \left[\widehat{w}_{j,j+\widehat{x}} f_j^\dagger f_{j+\widehat{x}} + \widehat{w}_{j,j+\widehat{y}} f_j^\dagger f_{j+\widehat{y}} + \text{ h.c} \right]$$

$$+ 2i\gamma \sum_j f_j^\dagger f_j$$

Complex fermion representation reveals steady-state solutions



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- Steady-state solution given by f_j vacuum
- Symmetry analysis implies only non-zero expectation values are W_i operators
- Fermion excitations cost energy 2γ, gauge-invariant 4γ excitation defines "fermion Liouvillian gap"

Analysis of "interlayer" gauge excitations

Flip intralayer gauge field at site k, Lindblad equation is now

$$i\mathcal{L} = J\sum_{\langle jk\rangle}(f_j^\dagger f_k + \text{ h.c}) + 2i\gamma\sum_{j\neq k} f_j^\dagger f_j + 2i\gamma(1 - f_k^\dagger f_k)$$

$\gamma \to \infty$, steady-state solution

$n_k = 1\,, n_{j\neq k} = 0$

Quantum Zeno effect - for strong dissipation, steady-state solutions emerge with definite Γ_k^5 eigenvalue

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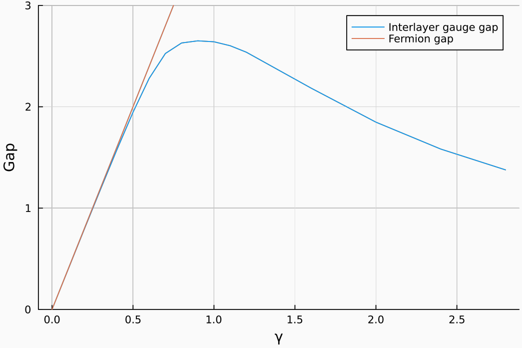

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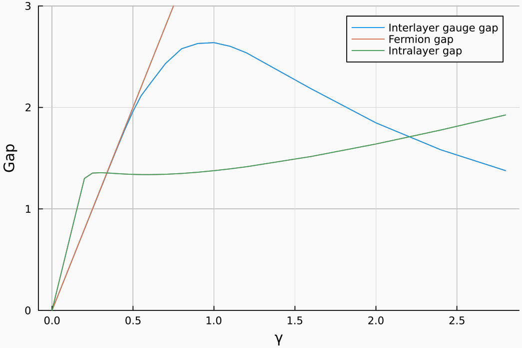
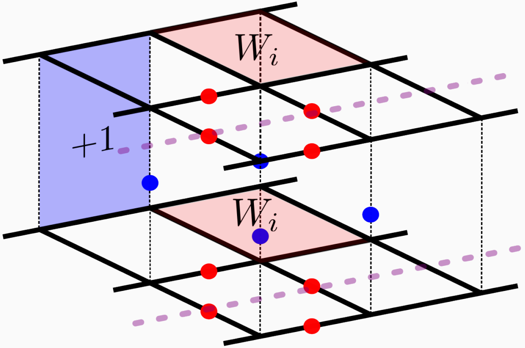
Analysis of “intralayer” gauge excitations

$$i\mathcal{L} = J \sum_{\langle jk \rangle \neq \langle j'k' \rangle} (f_j^\dagger f_k + \text{ h.c}) + J(f_{j'}^\dagger f_{k'}^\dagger + \text{ h.c}) + 2i\gamma \sum_j f_j^\dagger f_j$$



- $W_{iR} \neq W_{iL}$ induces pairing terms, symmetry sector corresponds to $\Gamma^{1,2,3,4}$ operators
- Gauge invariance prevents Zeno effect as $\gamma \to \infty$

What about topological order?



Four degenerate steady-states - “topological order” in doubled Hilbert space, but quantum superpositions are classical in our original Hilbert space

General picture: gauge sectors define distinct equilibration timescasles