

# Fractionalized Fermi Liquids: Mean-Field Theories, Instabilities, and Variational Wavefunctions

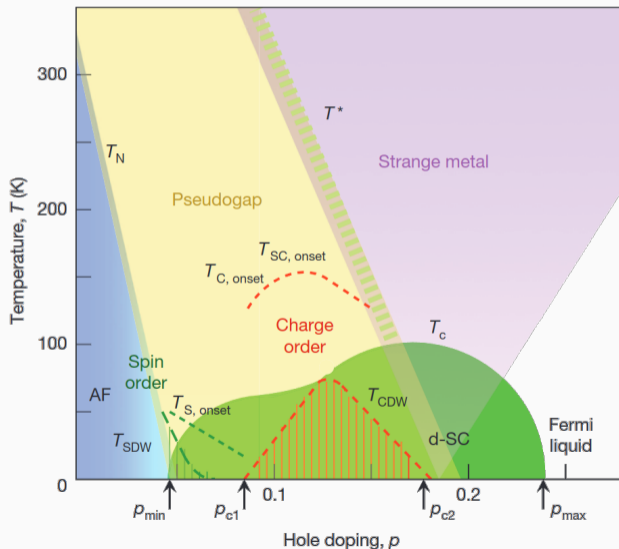
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Henry Shackleton

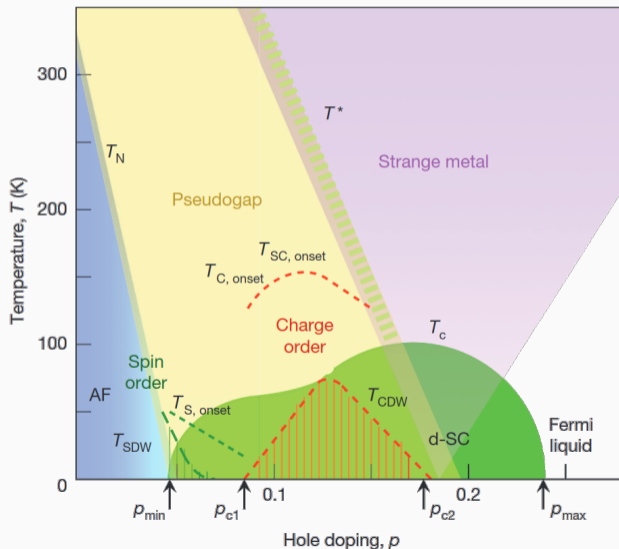
June 18, 2024



# Cuprate phase diagram as an inspiration for correlated physics

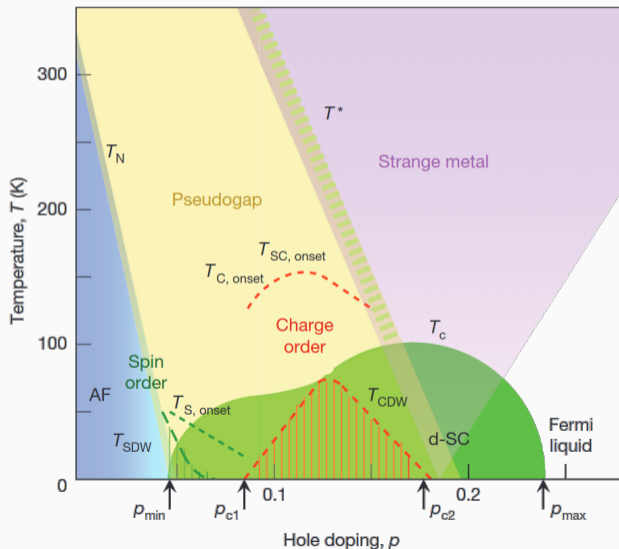


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- “Strange metal” phase -  $T$  linear resistivity from a theory of a quantum critical metal

# Cuprate phase diagram as an inspiration for correlated physics



- “Strange metal” phase -  $T$  linear resistivity from a theory of a quantum critical metal
- Pseudogap metal and proximate ordered phases from a theory of fractionalized Fermi liquids

## (Partial) acknowledgements



Subir Sachdev



Yahui Zhang



Maria  
Tikhanovskaya



Jonas von  
Milczewski



Dirk Morr



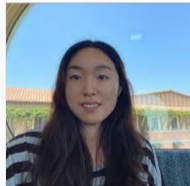
Darshan Joshi



Maine Christos



Alexander  
Nikolaenko



Zhu-Xi Luo



Eric Mascot

## Parton construction for fractionalized insulators: quantum spin liquids

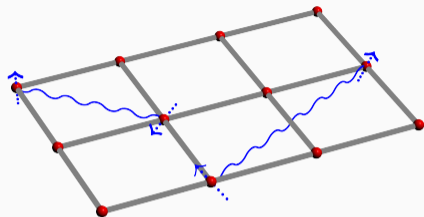
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$\vec{S}_i \rightarrow f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$ , emergent gauge fluctuations

$$H \rightarrow \sum_{ij} t_{ij} f_{i\sigma}^\dagger f_{j\sigma} + \dots$$



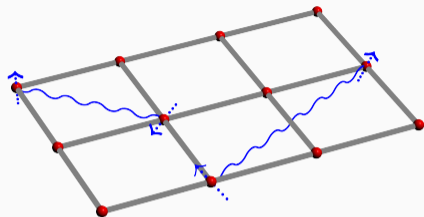
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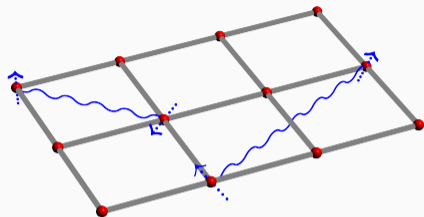


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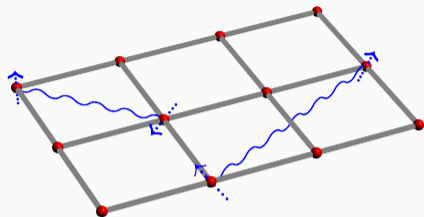
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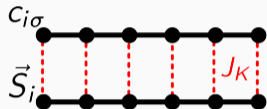
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- Bosonic/fermionic theories, classification with projective symmetry groups
- Instabilities to ordered phases (spinon condensation, confining instabilities)
- Numerical evaluation of correlated wavefunctions,  $\mathcal{P}_G |\psi_0\rangle$  - important for quantitative predictions

# Fractionalized Fermi liquids in Kondo lattices<sup>1</sup>

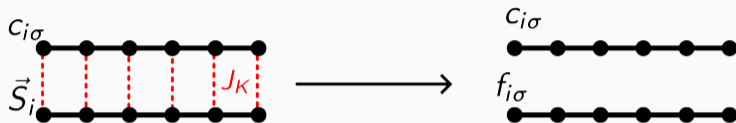
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<sup>1</sup>Senthil, Sachdev, and Vojta, *Physical Review Letters*, 2003.

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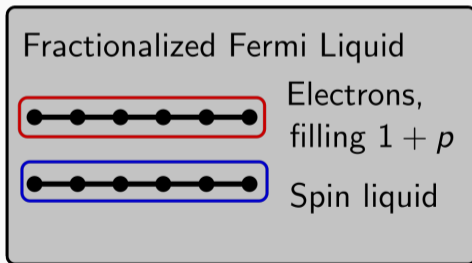
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Fractionalized Fermi Liquid

Electrons,  
filling  $1 + p$

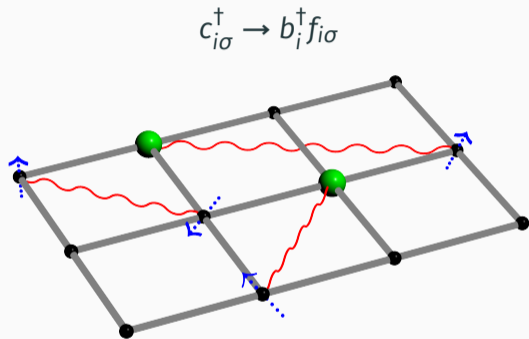
Spin liquid

Heavy Fermi Liquid

Electrons, filling  $p$

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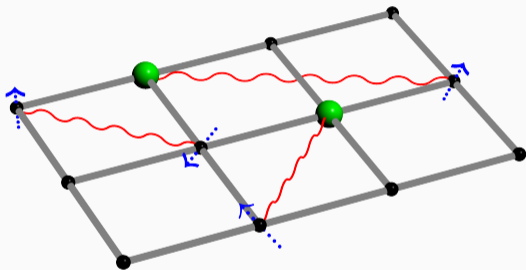
## Conventional electron fractionalization for single-band models<sup>2</sup>



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$$c_{i\sigma}^\dagger \rightarrow b_i^\dagger f_{i\sigma}$$

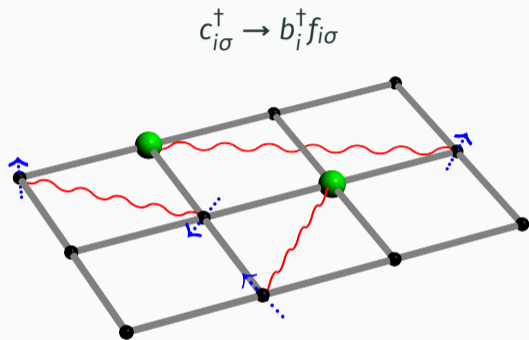


- Electron-like excitations given by spinon/holon bound state

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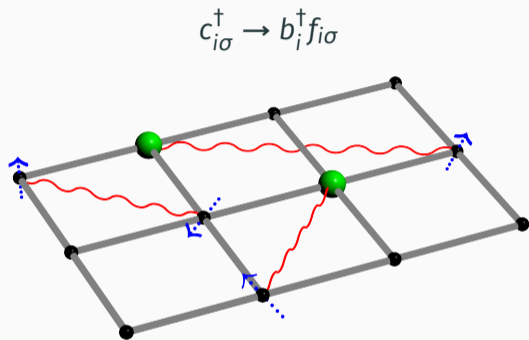
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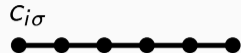
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- Electron-like excitations given by spinon/holon bound state
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- Obstacles to constructing correlated wavefunctions

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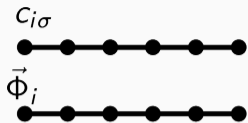
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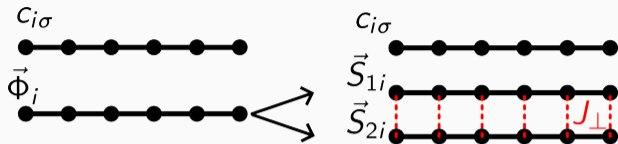
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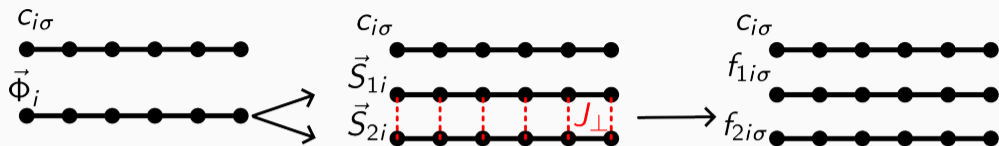
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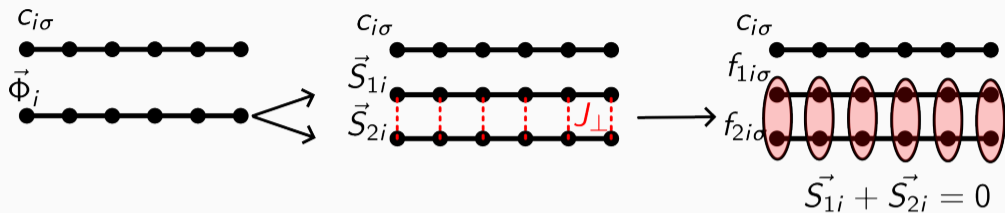
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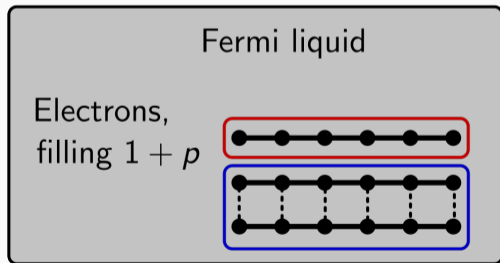
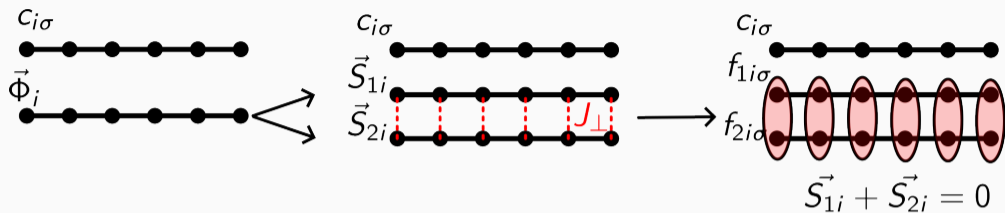
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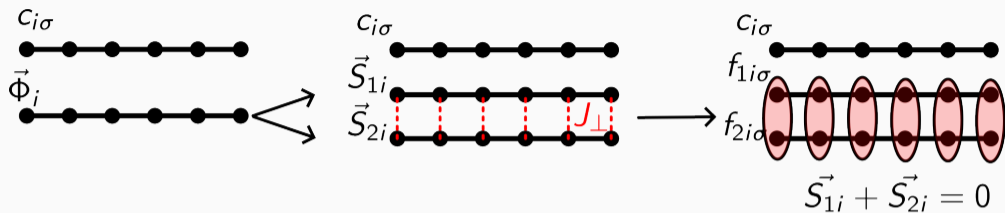
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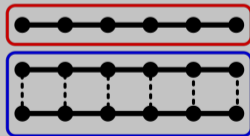


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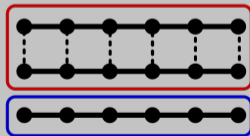
## Fermi liquid

Electrons,  
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## Fractionalized Fermi liquid

Electrons, filling  $p$



Spin liquid

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## Mean-field analysis on square lattice yields pseudogap-like features

Mean-field picture: electron-like quasiparticles + decoupled spin liquid <sup>4</sup>

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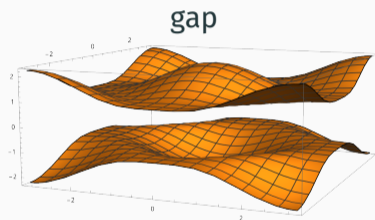
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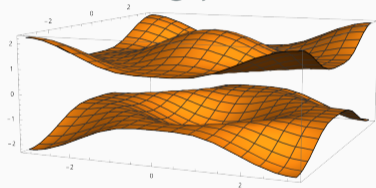
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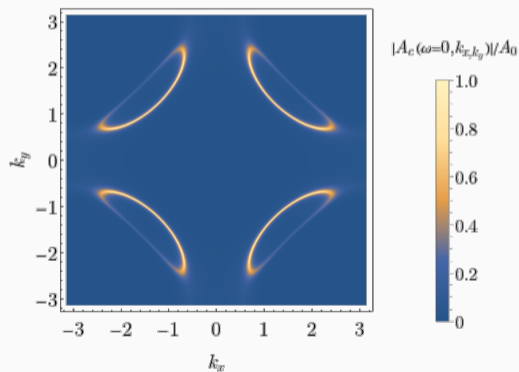
Half filling: Mott insulator with charge

gap



Hole doping similar to YRZ ansatz for

Green's function<sup>5</sup>



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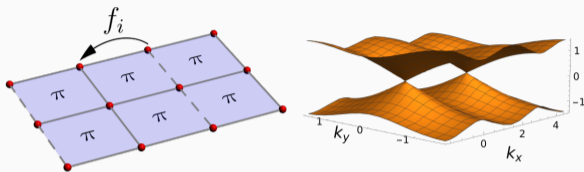
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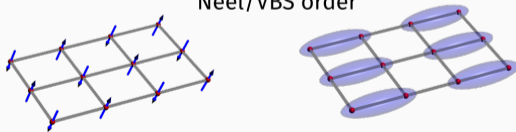
# Choice of spin liquid dictates proximate phases

- Intrinsic instabilities in spin liquid phase give one route to ordered phases
- Fermionic theory of a  $\pi$ -flux spin liquid leads to Néel/VBS order <sup>6</sup>



$N_f = 2$  QCD<sub>3</sub>, emergent SO(5) symmetry

Unstable to  
Néel/VBS order

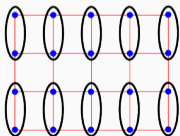
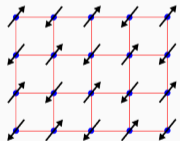


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# Charge instabilities arise from chargon condensation

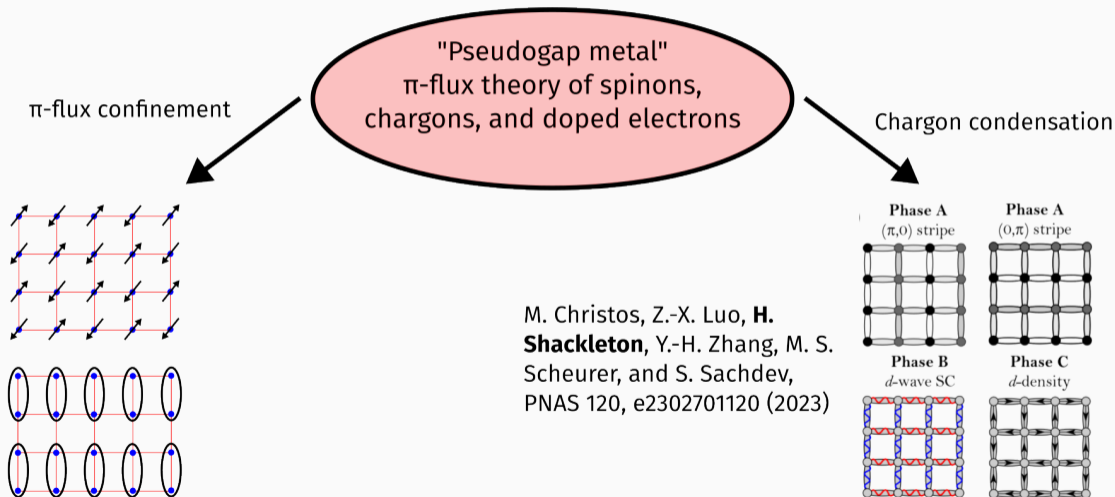
$\pi$ -flux confinement

"Pseudogap metal"  
 $\pi$ -flux theory of spinons,  
chargons, and doped electrons



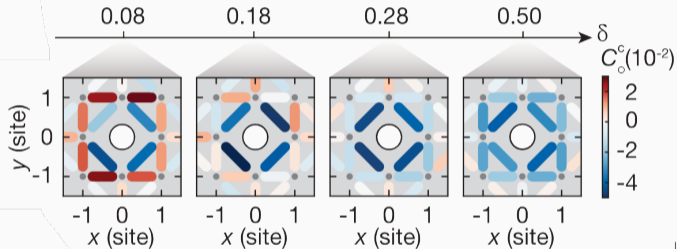
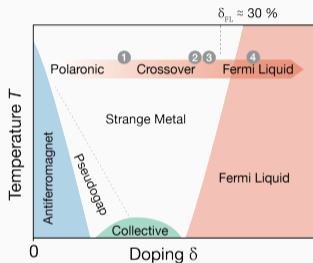


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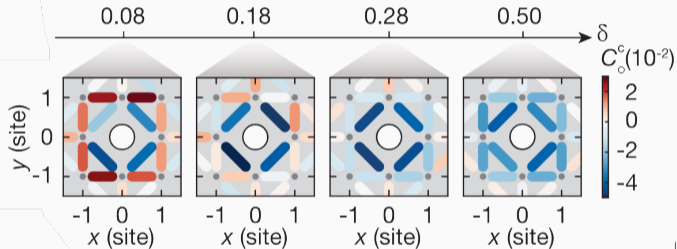
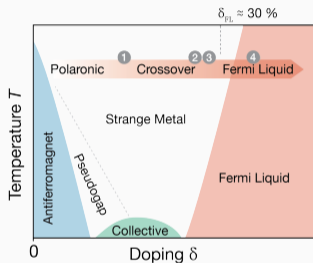
# Polaronic correlations central for capturing doped Mott insulators

$$H = t \sum_{\langle ij \rangle} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



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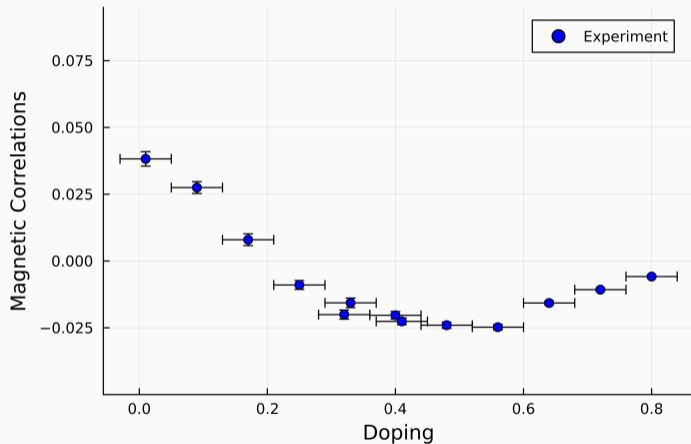
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Do these wavefunctions support polaronic correlations?

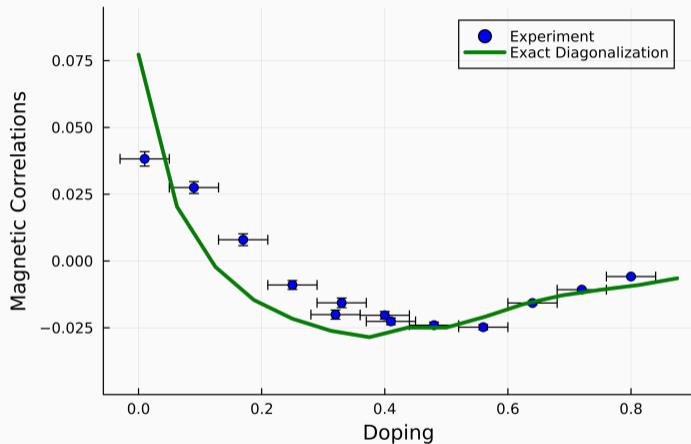
# Nearest neighbor magnetic correlations ( $U/t = 7.4$ )

Polaronic correlations probed by multipoint correlator  $\langle h_i S_j^z S_k^z \rangle$



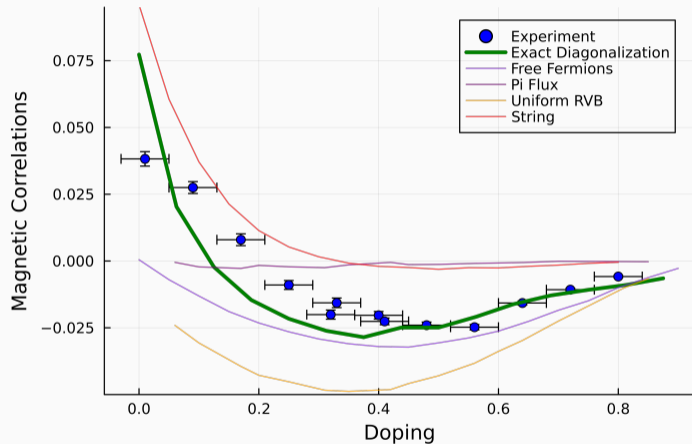
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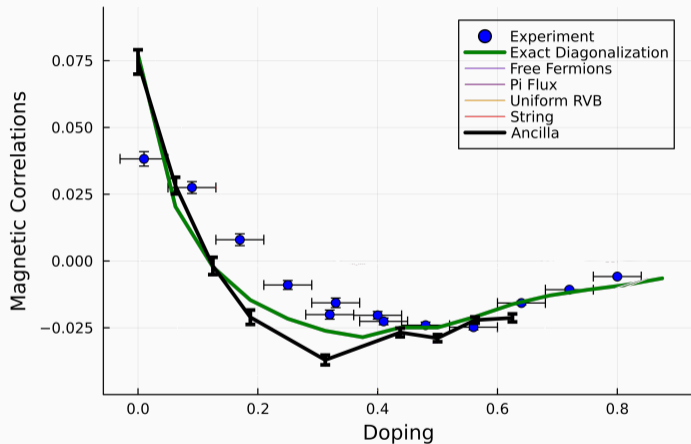
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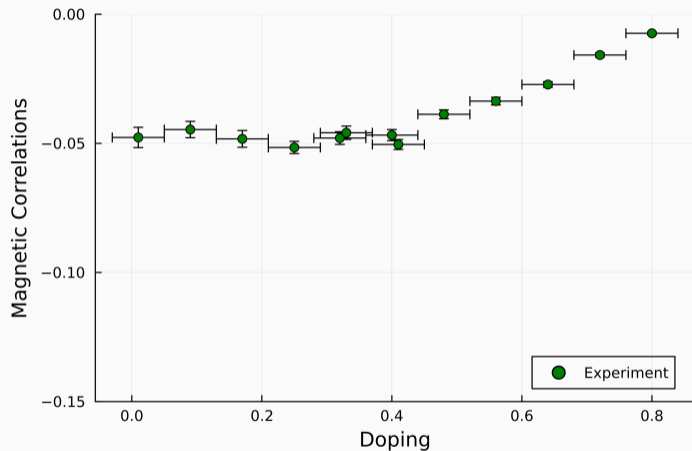


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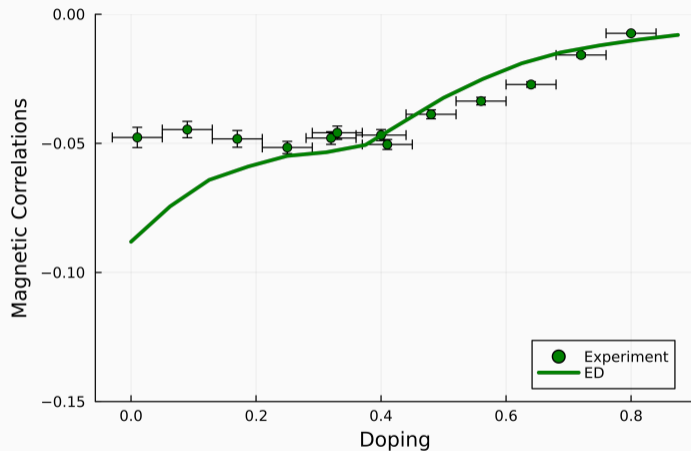


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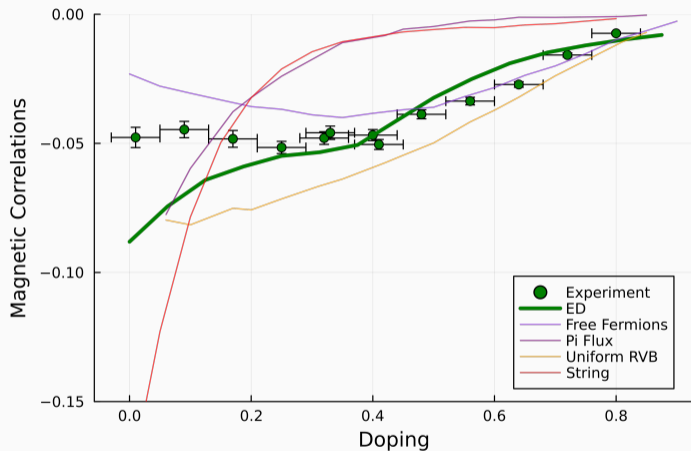




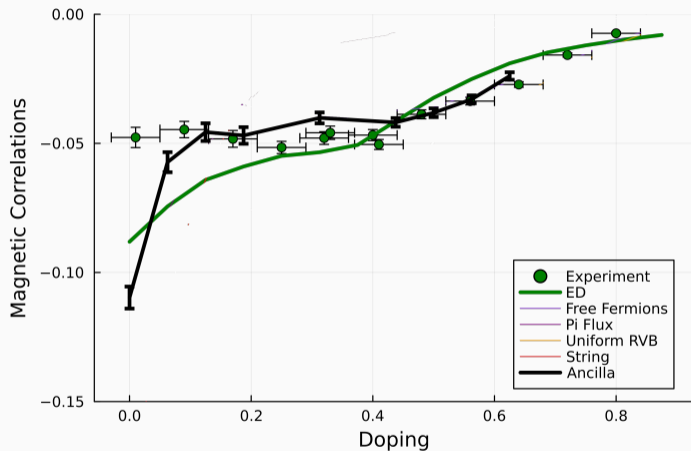
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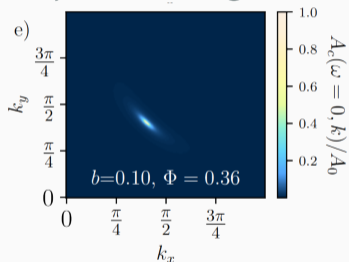


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## Related work and future directions

### Nodal anisotropic quasiparticles in superconducting state <sup>7</sup>



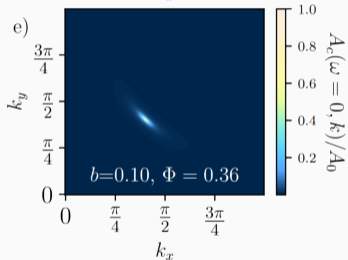
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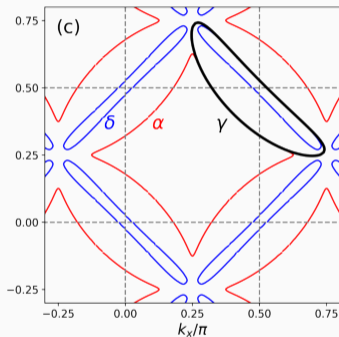
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FS reconstruction in CDW<sup>8</sup>  
 $FL^* \rightarrow CDW$



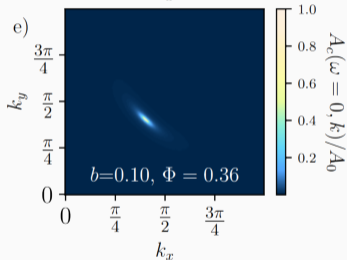
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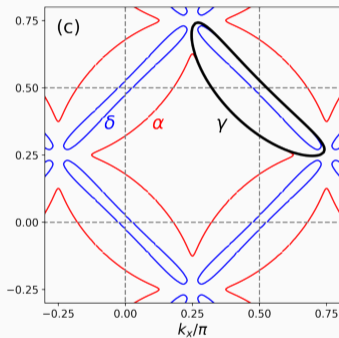
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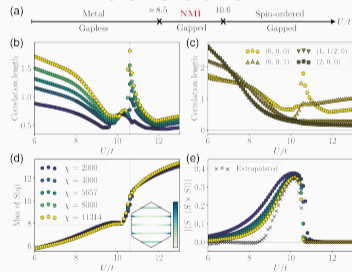
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Variational wavefunctions for spin liquids emerging at metal/insulator transitions<sup>9</sup>



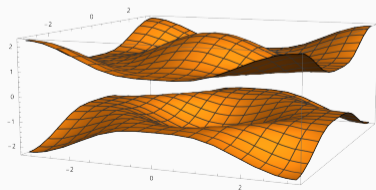
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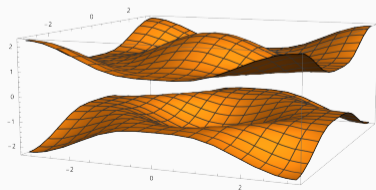
## Half-filling: wavefunctions behave favorably energetically

- Actual ground state: AF insulator for  $U/t > 0$
- $\pi$ -flux spin liquid gives low-energy variational ansatz in the Heisenberg limit
- Simple ansatz gives mean-field charge gap  $2\Phi$ , which we fix to be  $U$



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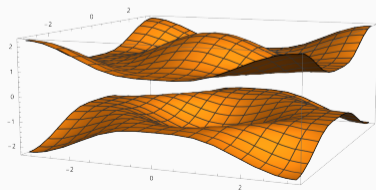
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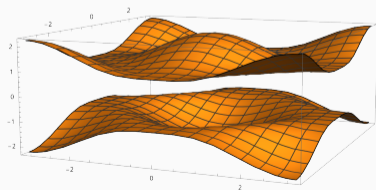
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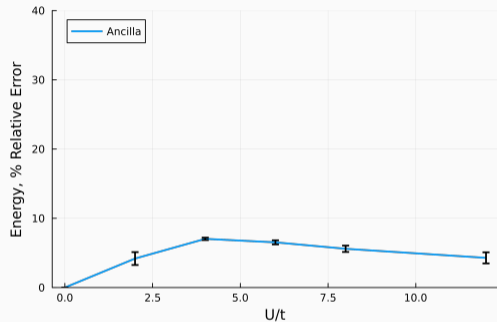
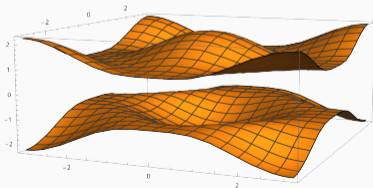
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