Protection of parity-time symmetry in topological many-body systems

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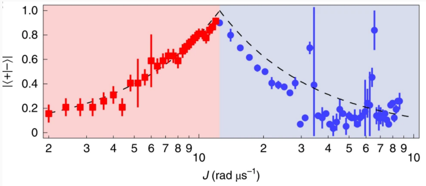
Non-Hermiticity Enriches Phase Diagrams

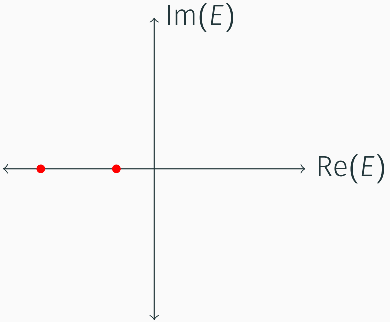
Non-Hermitian extensions yield

- Exotic phase transitions
- Unique topological invariants/phases
- Exceptional points



Pseudo-Hermiticity/$\mathcal{PT}$-symmetry

$\eta H = H^\dagger \eta$ implies $E^* \leftrightarrow E$



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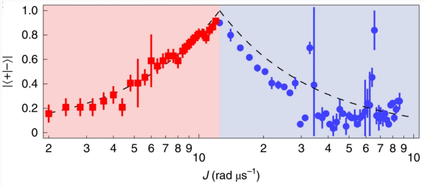

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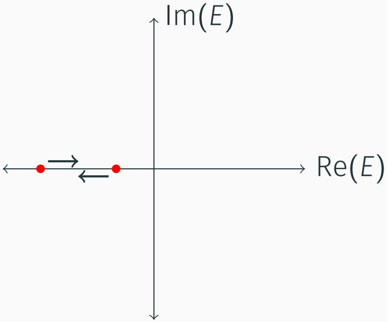

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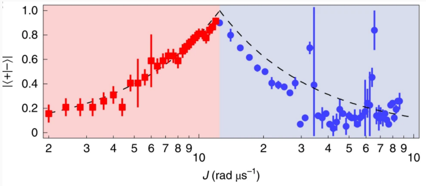

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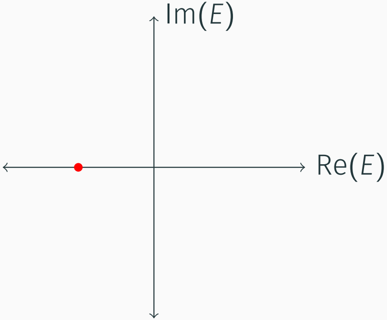

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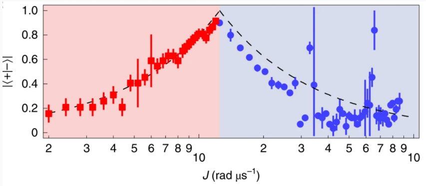

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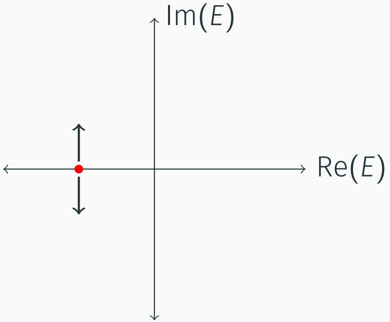

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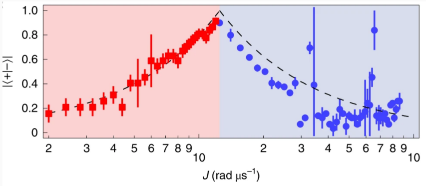

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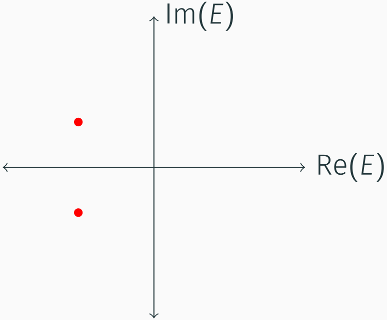

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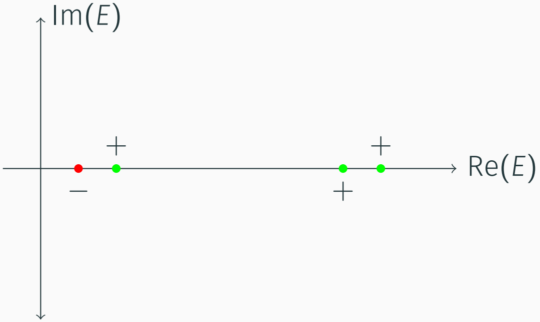
Symmetry-Based Criteria for Exceptional Points



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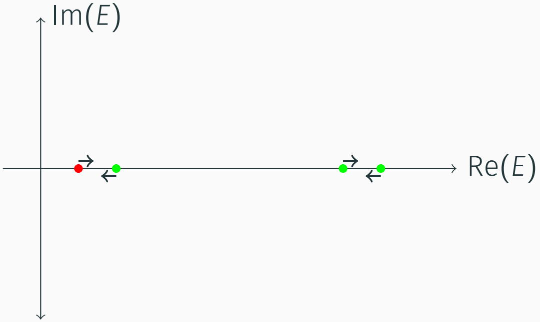

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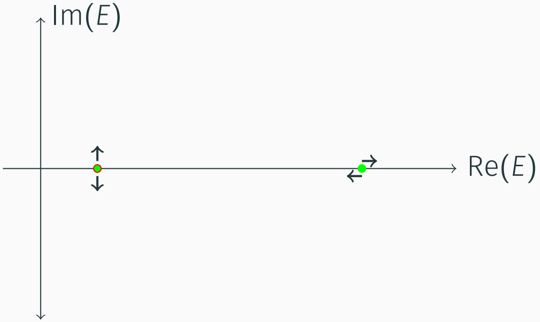

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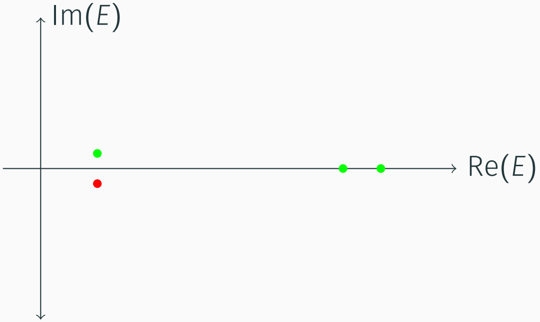

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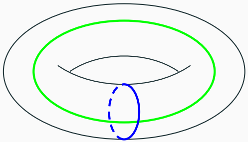
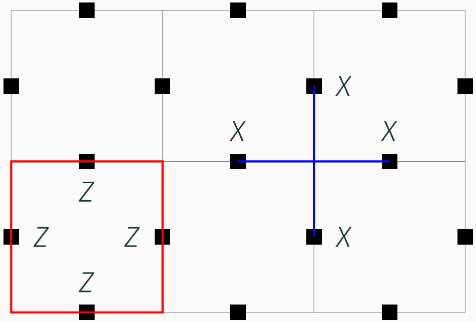
Topological GSD Not Symmetry-Based, May Yield Non-Trivial Physics



GSD arises from topological properties; no reason any additional symmetries should act non-trivially

Choice of symmetry dictates the form of non-Hermitian perturbations

Stabilizer Codes Yield Unique Symmetries



$$H = -\sum_c A_c - \sum_p B_p$$

Unique class of symmetries that allows disordered perturbations on entire lattice

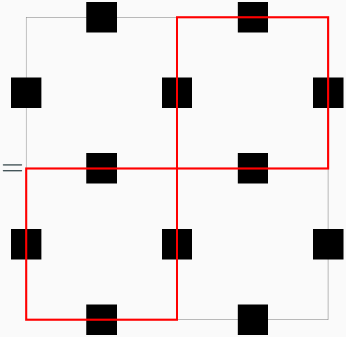
$$\eta = \prod_i X_i\,, Y_i\,, Z_i$$

Allows $i\sum_i g_i Z_i$, $i\sum_{ijk} g_{ijk} X_i Z_j Z_k$...

Kitaev, "Fault-tolerant quantum computation by anyons".

Geometric Criteria for Protection of Code Space

$\eta = \prod_i Z_i =$

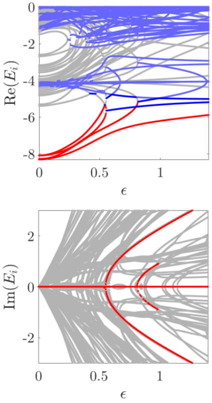


Even-by-even toric code has reality-protected GSD to generic psuedo-Hermitian perturbations, other system sizes unstable

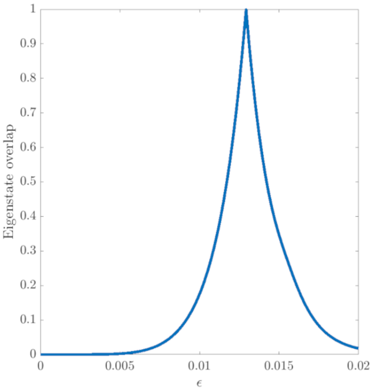
Exceptional Points Arise In Odd System Sizes

$$H = H_{TC} + \sum_i g_i X_i + i\epsilon \sum_i g_i' Z_i$$

Even-by-even



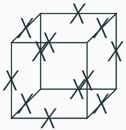
Odd-by-even

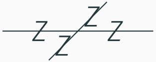


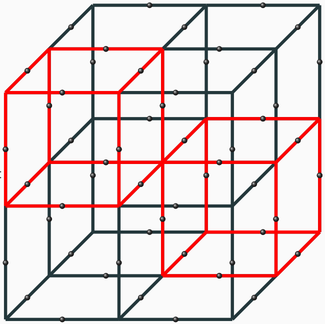
Geometric Criteria Generalizes to Fracton Models

X-cube model GSD protected on $e \times e \times e$ lattice

$$H = A_v + \sum_{i=x,y,z} B_p^i$$

$A_v =$ 

$B_p^i =$ 

$\prod_i X_i =$ 

Vijay, Haah, and Fu, "Fracton topological order, generalized lattice gauge theory, and duality".

Geometric Criteria Generalizes to Fracton Models

- Checkerboard model[1] always works (only definable on even system sizes)
- Quantum fractal spin liquids[2] also protected
- Haah's cubic codes[3] gives diverse classification[4]

System Size	CC_1	CC_2 CC_3 CC_6 CC_9	CC_4	CC_5 CC_8 CC_{10} CC_{16}	CC_7	CC_{11} CC_{12} CC_{14} CC_{15}	CC_{13}	CC_{17}
E $\times$ E $\times$ E E $\times$ E $\times$ e	✓	✓	✓	✓	✓	✓	✓	✓
e $\times$ E $\times$ E o $\times$ E $\times$ E E $\times$ e $\times$ E E $\times$ e $\times$ e	✓	✓	✓	✓	✓	✓	✓	✗
e $\times$ e $\times$ E e $\times$ E $\times$ E E $\times$ o $\times$ E e $\times$ e $\times$ e	✓	✓	✓	✓	✓	✓	✗	✗
E $\times$ E $\times$ o	✓	✓	✓	✓	✓	✗	✗	✓
e $\times$ o $\times$ E E $\times$ o $\times$ e e $\times$ o $\times$ e	✗	✓	✓	✓	✓	✓	✗	✗
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o $\times$ o $\times$ E o $\times$ o $\times$ e	✗	✗	✗	✗	✓	✓	✗	✗
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E × E × E E × E × e	✓	✓	✓	✓	✓	✓	✓	✓
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Conclusions

- Topological order provides a unique setting for studying non-Hermitian perturbations
- Stabilizer models naturally give rise to system size-dependent behavior of psuedo-Hermitian perturbations
- Open questions: what is the nature of topological order at an exceptional point?