A sign-problem-free effective model of triangular lattice antiferromagnetism

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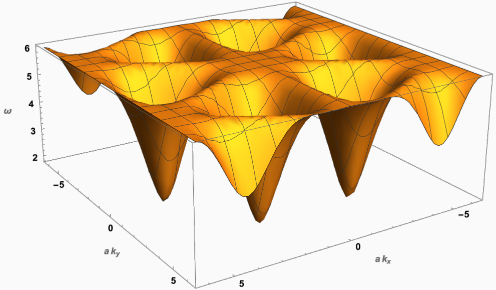
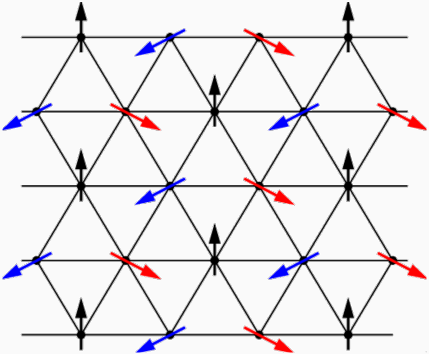
Frustrated antiferromagnetism leads to exotic quantum phases

- $H = \sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$, trivial phase for half-integer spin precluded by LSM theorem
- Study *effective* model starting from bosonic spinon construction, $\mathbf{S}_i = b^\dagger_{i\alpha}\boldsymbol{\sigma}^{\alpha\beta}b_{i\beta}$





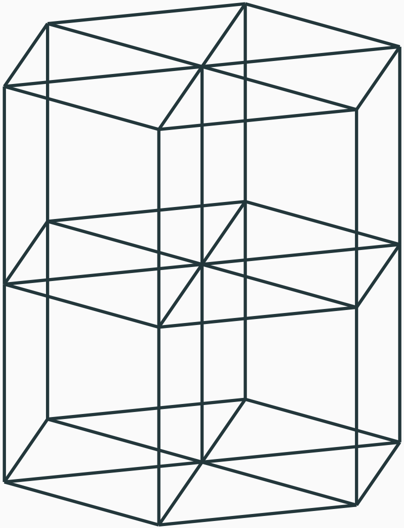
Sachdev, "Kagome- and Triangular-Lattice Heisenberg Antiferromagnets".

Effective model of triangular lattice antiferromagnetism

O(4) bosonic spinons $z_{i\alpha}$, $\alpha = 1\,,2$

coupled to an *odd* $\mathbb{Z}_2$ gauge field $s_{j,j+\widehat{\mu}}$

$$H[z_\alpha, s] = -\frac{J}{2}\sum_{\langle j,\mu\rangle} s_{j,j+\widehat{\mu}}\left(z^*_{j,\alpha} z_{j+\widehat{\mu},\alpha} + \text{ c.c}\right) - K\sum_{\triangle\square}\prod_{\triangle\square} s_{j,j+\widehat{\mu}} + \underbrace{i\pi\sum_j s_{j,j+\widehat{\tau}}}_{\text{Berry phase}}$$



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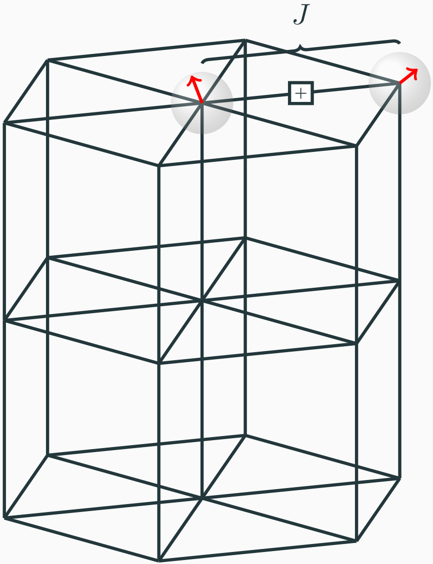

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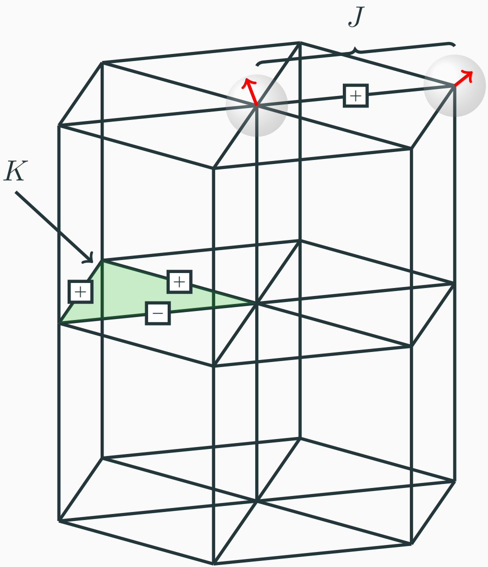

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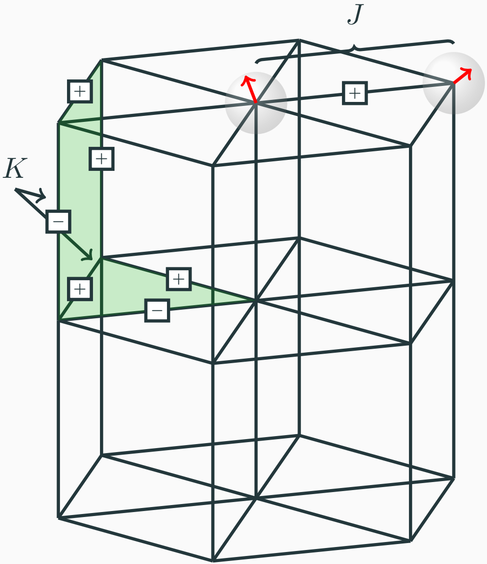

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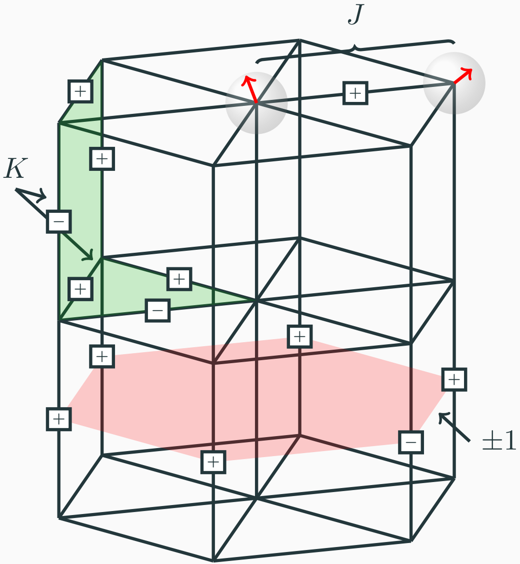

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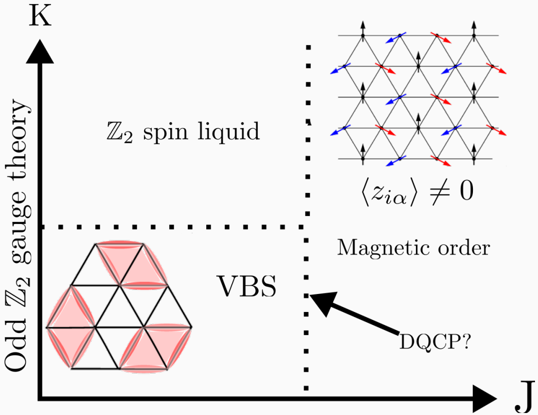

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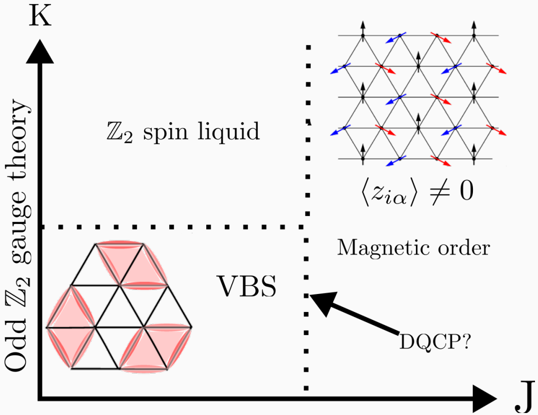


Session D54: Quantum Spin Liquids: Theory and Computation
3:00 PM–5:48 PM, Monday, March 6, 2023
Room: Room 306

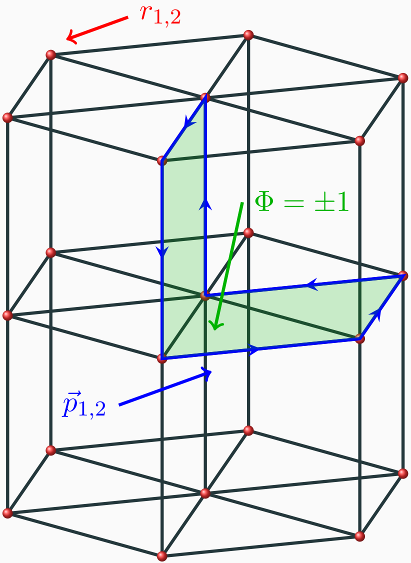
Abstract: D54.00007 : Quantum Electrodynamics in 2+1 Dimensions as the Organizing Principle of Triangular Lattice Antiferromagnets
4:12 PM–4:24 PM

← *Abstract* →

Presenter:
Andreas M Läuchli
(Paul Scherrer Institute)

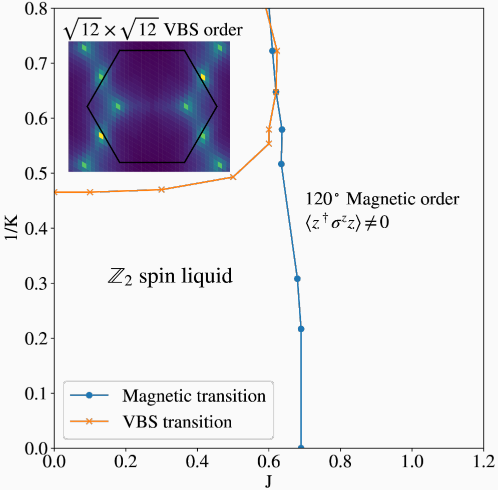
Berry phase requires sign-problem-free mapping

- $z_{j,\alpha} = r_{j,\alpha} e^{i\theta_{j,\alpha}}$, angular variables recast into closed current loops
- Non-trivial constraint between allowed gauge flux Φ and current loops
- Berry phase reflected in *geometric frustration* of gauge flux



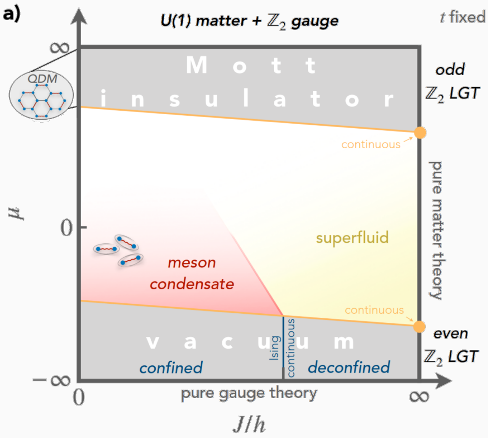
Phase diagram captures spin liquid and ordered phases

- Preliminary data indicates direct transition between VBS and magnetic order
- System sizes restricted to multiples of 12 to admit VBS order
- Independent spinon/gauge cluster updates insufficient to resolve transition at large system sizes



Future work

- Improved global updates for studying DQCP - can we find signatures of QED_3?
- Generalization to continuous-time simulations
- Application of sign-problem-free mapping to more systems - Bose Hubbard + $\mathbb{Z}_2$ at finite density



Homeier et al., *Quantum Simulation of Z2 Lattice Gauge Theories with Dynamical Matter from Two-Body Interactions in (2+1)D*.

Full sign-problem-free Hamiltonian

$$\mathcal{Z} = \sum_{h_{\bar{j},\alpha,\mu}=-\infty}^{\infty} \prod_{j\alpha} \int_0^1 r_{j,\alpha}\, \mathrm{d}r_{j,\alpha}\, \delta\left(\sum_{\alpha} r_{j,\alpha}^2 - 1\right) \exp\left(-H[r_\alpha, h_\alpha]\right)$$

$$H[r_\alpha, h_\alpha] = \sum_{\langle j,\mu\rangle} \left[-\ln I_{p_{j,\alpha,\mu}}(J r_{j,\alpha} r_{j+\widehat{\mu},\alpha}) + K_d \varepsilon_{\bar{j},\mu} \sigma_{\bar{j},\mu}\right]$$

$$\sigma_{\bar{j},\mu} = 2 \sum_{\alpha} h_{\bar{j},\alpha,\mu} \mod 2 - 1$$

$$e^{2K_d} = \tanh K = \frac{e^K - e^{-K}}{e^K + e^{-K}}$$