Conductance and thermopower fluctuations in interacting quantum dots

Henry Shackleton

March 7, 2024

Harvard University

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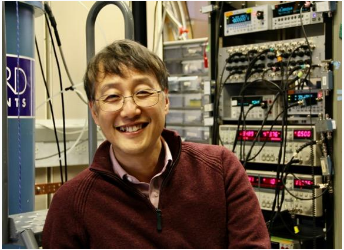
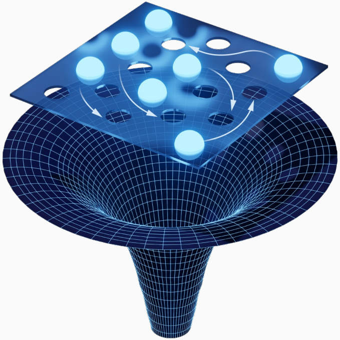



w/ Laurel Anderson, Philip Kim, and Subir Sachdev, arXiv:2309.05741

SYK as a minimal model for holographic physics

$$H = \frac{1}{(2N)^{\frac{3}{2}}} \sum_{ijkl} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l \qquad \langle J_{ij;kl} \rangle = 0 \quad \langle J^*_{ij;kl} J_{ij;kl} \rangle = J^2$$



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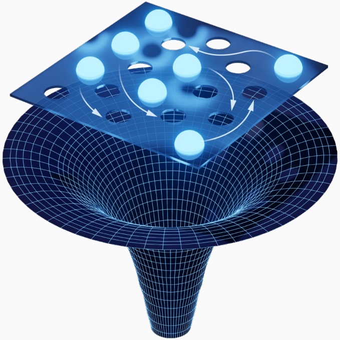

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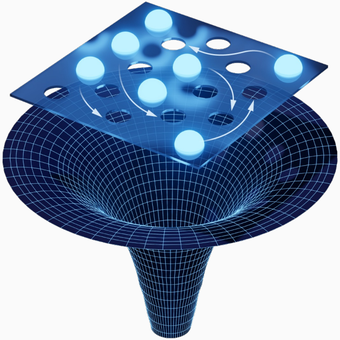

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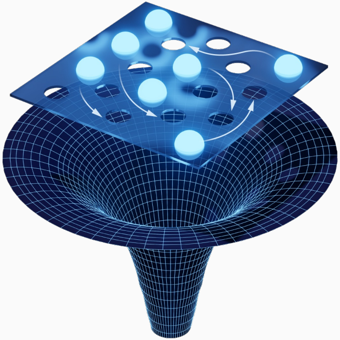

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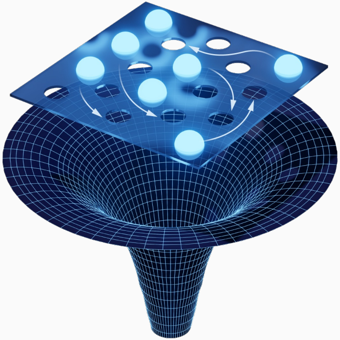

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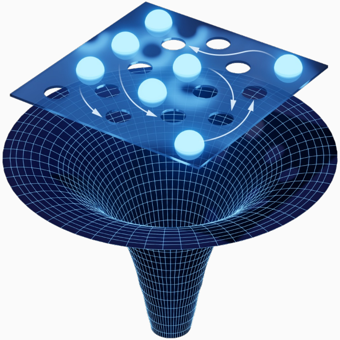

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Experimental challenges: suppress kinetic energy, generate disordered interactions

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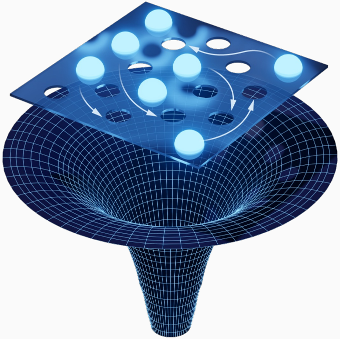
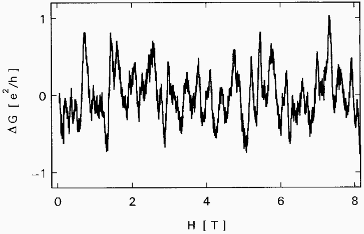


Can we use sample-to-sample fluctuations as a diagnostic of strongly correlated physics?

Transport fluctuations as a probe of non-Fermi liquid physics

Fermi liquid

- Universal conductance fluctuations from single-particle chaos[1]



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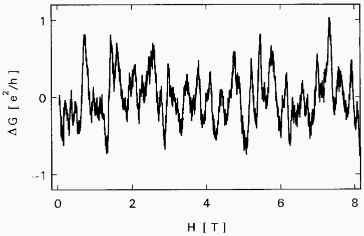

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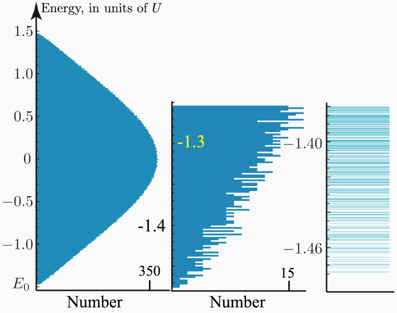
SYK model

- Exponential DOS at low energy



- Strongly self-averaging

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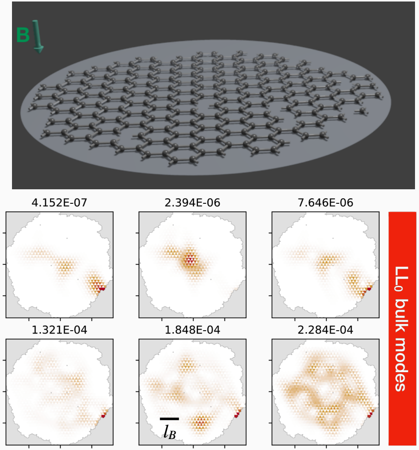
Proposed realizations in disordered graphene flakes[3]



[2] Anderson et al., arXiv:2401.08050, 2024

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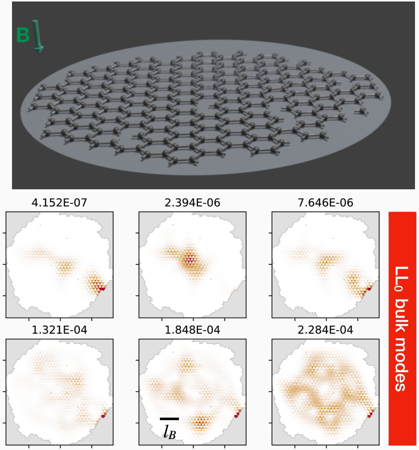
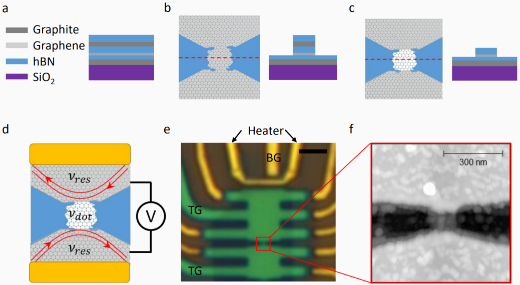


Exp: Laurel Anderson (W06.003)[2]



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Non-Fermi liquid physics probed through transport quantities

Competing energy scales:

- SYK interaction J
- Random hopping t
- Charging energy E_c
- Coupling to leads Γ
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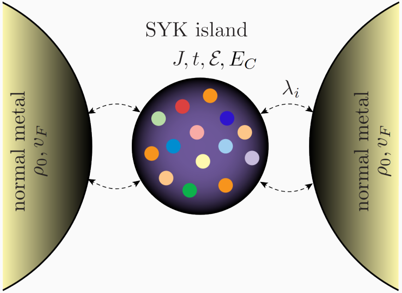

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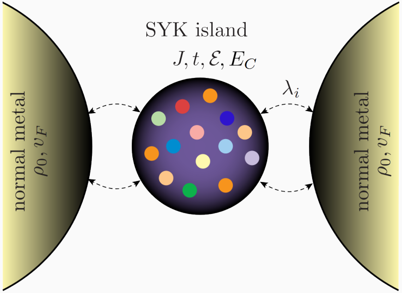

This talk: consider competition between Fermi liquid (t) and SYK physics (J)

$$H = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{N^{1/2}} \sum_{ij} t_{ij} c_i^\dagger c_j + \mu \sum_i c_i^\dagger c_i$$

$$\langle J_{ij;kl} \rangle = \langle t_{ij} \rangle \quad \langle J_{ij;kl}^* J_{ij;kl} \rangle = J^2 \quad \langle t_{ij}^* t_{ij} \rangle = t^2$$

Transport quantities: disordered Fermi liquid below $E_{\mathsf{coh}} \sim t^2/J$, SYK above

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$$G(\omega) \sim \begin{cases} G_{\mathrm{FL}}(\omega) & T, \omega \ll E_{\mathrm{coh}} \quad (\text{with } t \to E_{\mathrm{coh}}) \\ G_{\mathrm{SYK}}(\omega) & T \gg E_{\mathrm{coh}} \end{cases}$$

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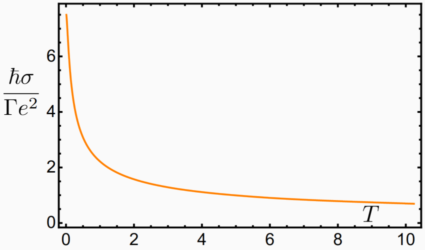
$$\sigma = \frac{2e^2\Gamma}{\pi\hbar} \int_{-\infty}^{\infty} d\omega f'(\omega) \, \mathrm{Im}\, G(\omega)$$

$$\sigma_{\mathrm{FL}} \sim \frac{e^2}{h} \frac{\Gamma}{E_{\mathrm{coh}}}$$

$$\sigma_{\mathrm{SYK}} \sim \frac{e^2}{h} \frac{\Gamma}{\sqrt{JT}}$$



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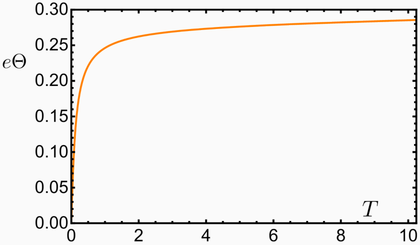
$$\Theta = \frac{1}{Te} \frac{\int_{-\infty}^{\infty} d\omega\, \omega f'(\omega) \operatorname{Im} G(\omega)}{\int_{-\infty}^{\infty} d\omega\, f'(\omega) \operatorname{Im} G(\omega)}$$

$$\Theta_{\text{FL}} \sim \frac{T}{e}$$

$$\Theta_{\text{SYK}} = \frac{4\pi}{3e}\mathcal{E}$$



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Try same philosophy for transport fluctuations: non-interacting fluctuations for $T \ll E_{\mathrm{coh}}$, SYK fluctuations for $T \gg E_{\mathrm{coh}}$

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Non-interacting Fermi liquid prediction: random matrix theory

Conductance fluctuations of a *closed* non-interacting quantum dot

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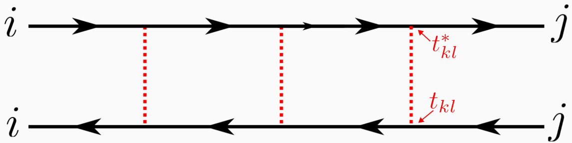

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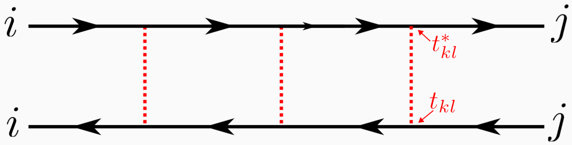

Pole at $|\omega - \epsilon|$, $T \to 0$, robust feature of FL

$$\mathrm{Var}\ \sigma \sim \mathrm{Var}\ \Theta \sim \frac{1}{NT}$$

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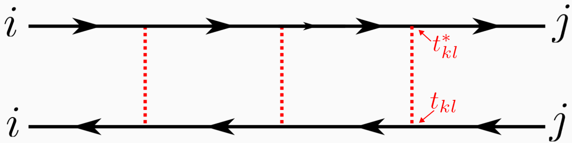

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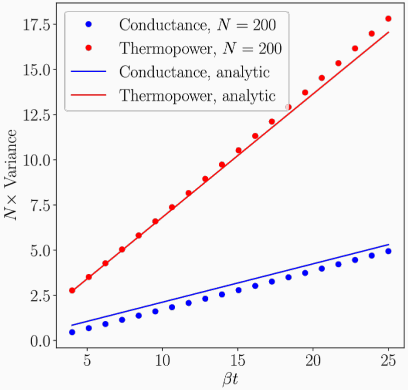
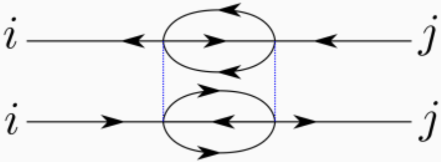


Pure SYK prediction: strongly self-averaging



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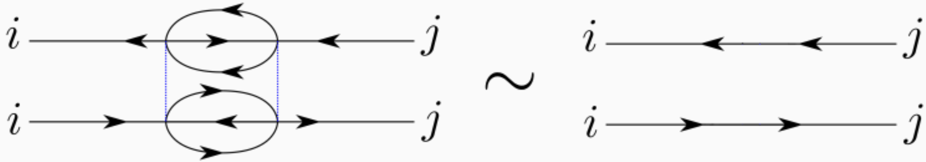

"Universal" fluctuations of spectral density

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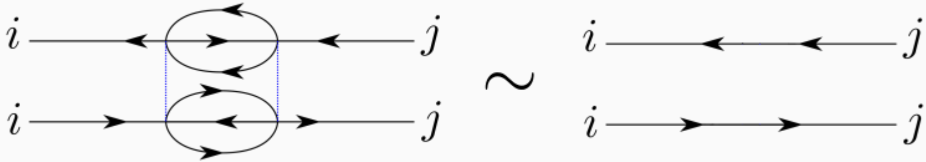

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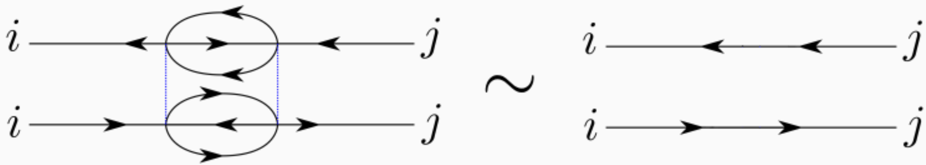

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Random hoppings still drive fluctuations even in SYK regime!

- How does a non-Fermi liquid “see” other forms of disorder?

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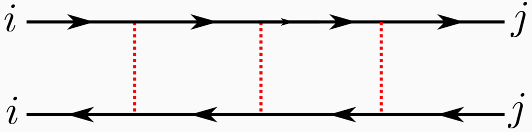

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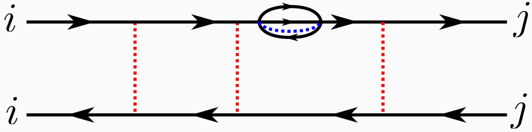

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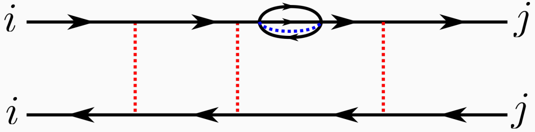

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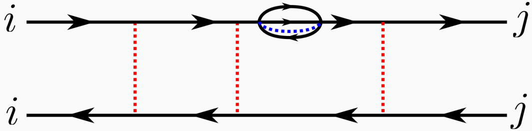

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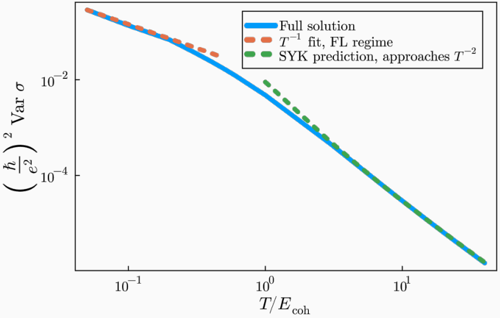

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- Strongly-interacting non-Fermi liquid can still “sense” single-particle disorder
- Non-universal suppression above E_{coh} driven by SYK physics

Connection to experiments: what are we measuring?

Actual experiments measure fluctuations from changing μ, B, etc

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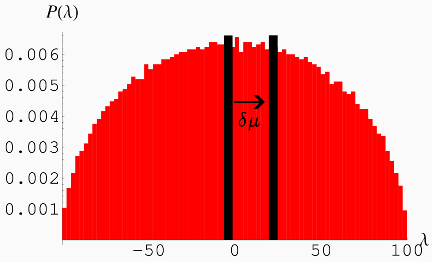
Non-interacting system: $\mu \to \mu + \delta\mu$

"re-draws" random matrix



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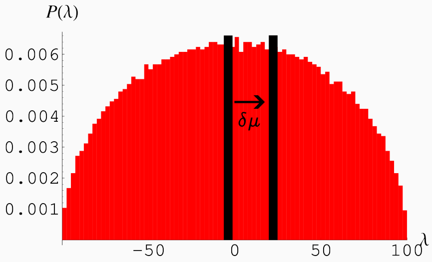
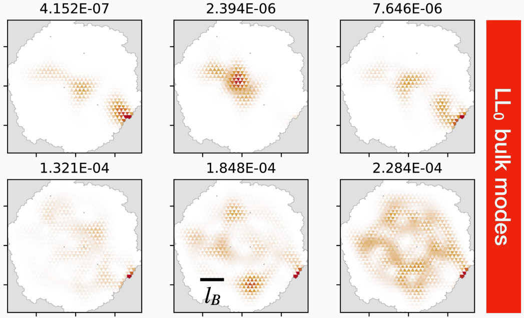


Graphene SYK setup - what all gets re-drawn?



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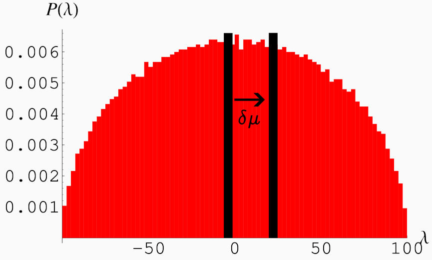

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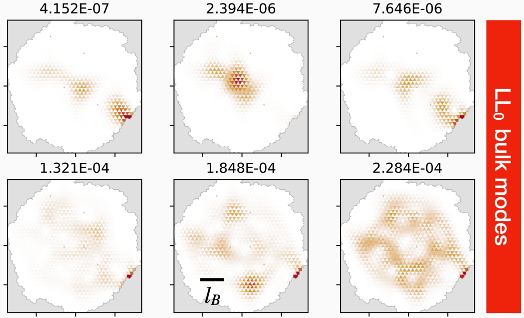
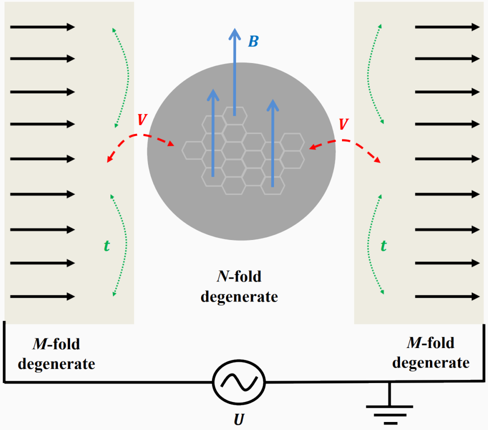


Full re-drawing is "worst case"

Future directions: Treatment for open quantum dot [5]



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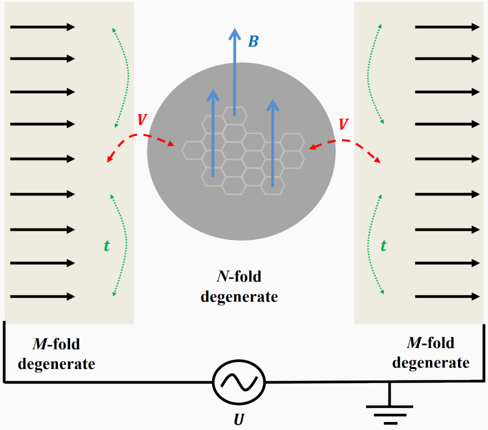
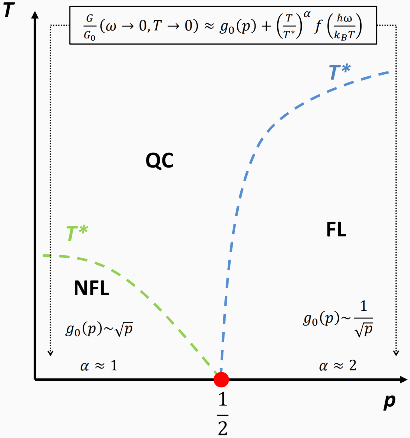
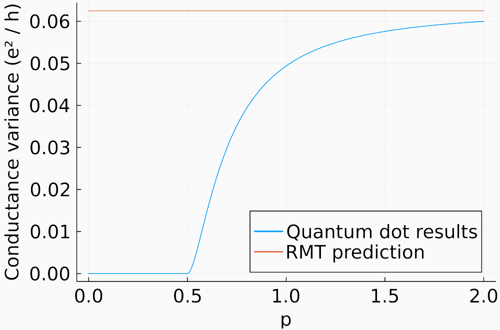
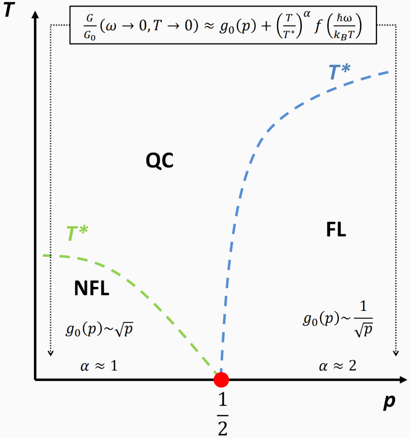




[5] Can, Nica, and Franz, *Physical Review B*,. 2019

Future directions: Treatment for open quantum dot [5]



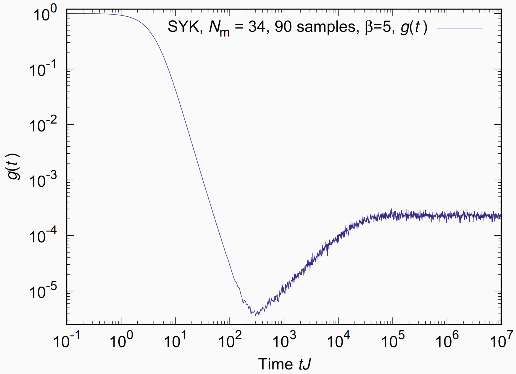


[5] Can, Nica, and Franz, *Physical Review B*,. 2019

Future directions: Observable signatures of *many-body* quantum chaos[6]

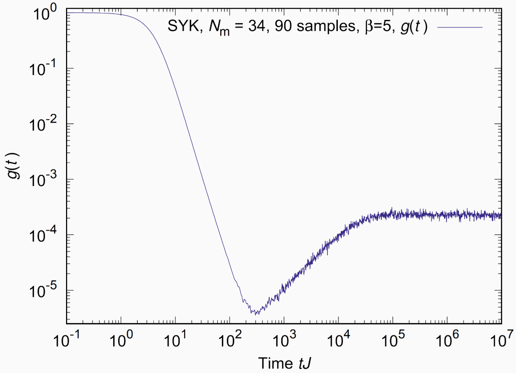
[6] Cotler et al., *Journal of High Energy Physics*, 2017.

Future directions: Observable signatures of *many-body* quantum chaos[6]

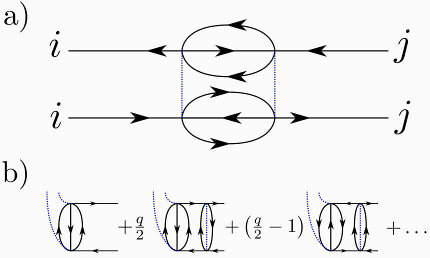


[6] Cotler et al., *Journal of High Energy Physics*, 2017.

Future directions: Observable signatures of *many-body* quantum chaos[6]



Can we find signatures of quantum chaos in single-particle observables?



[6] Cotler et al., *Journal of High Energy Physics*, 2017.

Thank you for your attention!