Sign-problem-free effective models for triangular lattice quantum antiferromagnets

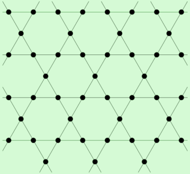
Henry Shackleton

November 15, 2023

Harvard University

Electron correlations for engineering macroscopic phenomena

Frustrated Magnetism



H. Shackleton and S. Zhang, in progress

H. Shackleton and S. Sachdev, arXiv:2311.01572 (2023)

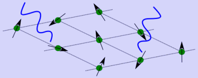
M. Christos, Z.-X. Luo, **H. Shackleton**, Y.-H. Zhang, M. S. Scheurer, and S. Sachdev, Proceedings of the National Academy of Sciences 120, e2302701120 (2023)

H. Shackleton and M. S. Scheurer, arXiv:2307.05743 (2023)

H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022, 141 (2022)

H. Shackleton, A. Thomson, and S. Sachdev, Phys. Rev. B 104, 045110 (2021).

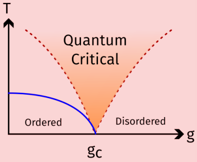
Non-Equilibrium Dynamics



H. Shackleton, L. E. Anderson, P. Kim, and S. Sachdev, arXiv:2309.05741 (2023)

H. Shackleton and M. S. Scheurer, Phys. Rev. Research 2, 033022 (2020)

Quantum Criticality

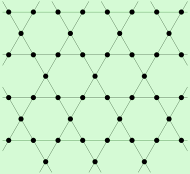


H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, Phys. Rev. Lett. 126, 136602 (2021)

Nivedita, **H. Shackleton**, and S. Sachdev, Phys. Rev. E 101, 042136 (2020)

Electron correlations for engineering macroscopic phenomena

Frustrated Magnetism



H. Shackleton and S. Zhang, in progress

H. Shackleton and S. Sachdev, arXiv:2311.01572 (2023)

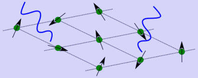
M. Christos, Z.-X. Luo, **H. Shackleton**, Y.-H. Zhang, M. S. Scheurer, and S. Sachdev, Proceedings of the National Academy of Sciences 120, e2302701120 (2023)

H. Shackleton and M. S. Scheurer, arXiv:2307.05743 (2023)

H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022, 141 (2022)

H. Shackleton, A. Thomson, and S. Sachdev, Phys. Rev. B 104, 045110 (2021).

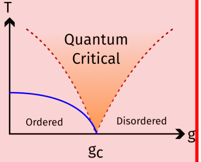
Non-Equilibrium Dynamics



H. Shackleton, L. E. Anderson, P. Kim, and S. Sachdev, arXiv:2309.05741 (2023)

H. Shackleton and M. S. Scheurer, Phys. Rev. Research 2, 033022 (2020)

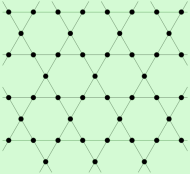
Quantum Criticality



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, Phys. Rev. Lett. 126, 136602 (2021)

Nivedita, **H. Shackleton**, and S. Sachdev, Phys. Rev. E 101, 042136 (2020)

Frustrated Magnetism



H. Shackleton and S. Zhang, in progress

H. Shackleton and S. Sachdev, arXiv:2311.01572 (2023)

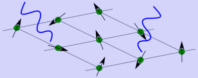
M. Christos, Z.-X. Luo, **H. Shackleton**, Y.-H. Zhang, M. S. Scheurer, and S. Sachdev, Proceedings of the National Academy of Sciences 120, e2302701120 (2023)

H. Shackleton and M. S. Scheurer, arXiv:2307.05743 (2023)

H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022, 141 (2022)

H. Shackleton, A. Thomson, and S. Sachdev, Phys. Rev. B 104, 045110 (2021).

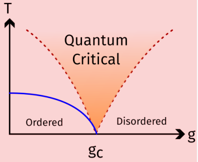
Non-Equilibrium Dynamics



H. Shackleton, L. E. Anderson, P. Kim, and S. Sachdev, arXiv:2309.05741 (2023)

H. Shackleton and M. S. Scheurer, Phys. Rev. Research 2, 033022 (2020)

Quantum Criticality

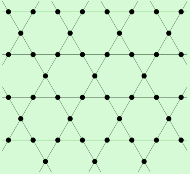


H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, Phys. Rev. Lett. 126, 136602 (2021)

Nivedita, **H. Shackleton**, and S. Sachdev, Phys. Rev. E 101, 042136 (2020)

Electron correlations for engineering macroscopic phenomena

Frustrated Magnetism



H. Shackleton and S. Zhang, in progress

H. Shackleton and S. Sachdev, arXiv:2311.01572 (2023)

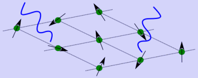
M. Christos, Z.-X. Luo, **H. Shackleton**, Y.-H. Zhang, M. S. Scheurer, and S. Sachdev, Proceedings of the National Academy of Sciences 120, e2302701120 (2023)

H. Shackleton and M. S. Scheurer, arXiv:2307.05743 (2023)

H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022, 141 (2022)

H. Shackleton, A. Thomson, and S. Sachdev, Phys. Rev. B 104, 045110 (2021).

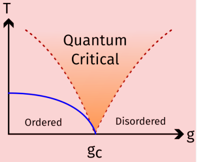
Non-Equilibrium Dynamics



H. Shackleton, L. E. Anderson, P. Kim, and S. Sachdev, arXiv:2309.05741 (2023)

H. Shackleton and M. S. Scheurer, Phys. Rev. Research 2, 033022 (2020)

Quantum Criticality

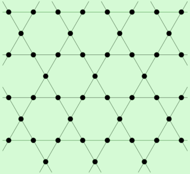


H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, Phys. Rev. Lett. 126, 136602 (2021)

Nivedita, **H. Shackleton**, and S. Sachdev, Phys. Rev. E 101, 042136 (2020)

Electron correlations for engineering macroscopic phenomena

Frustrated Magnetism



H. Shackleton and S. Zhang, in progress

H. Shackleton and S. Sachdev, arXiv:2311.01572 (2023)

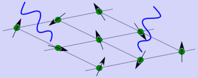
M. Christos, Z.-X. Luo, **H. Shackleton**, Y.-H. Zhang, M. S. Scheurer, and S. Sachdev, Proceedings of the National Academy of Sciences 120, e2302701120 (2023)

H. Shackleton and M. S. Scheurer, arXiv:2307.05743 (2023)

H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022, 141 (2022)

H. Shackleton, A. Thomson, and S. Sachdev, Phys. Rev. B 104, 045110 (2021).

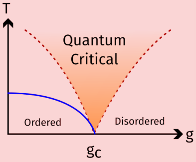
Non-Equilibrium Dynamics



H. Shackleton, L. E. Anderson, P. Kim, and S. Sachdev, arXiv:2309.05741 (2023)

H. Shackleton and M. S. Scheurer, Phys. Rev. Research 2, 033022 (2020)

Quantum Criticality



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, Phys. Rev. Lett. 126, 136602 (2021)

Nivedita, **H. Shackleton**, and S. Sachdev, Phys. Rev. E 101, 042136 (2020)

Sign-problem-free effective models for triangular lattice quantum antiferromagnets

Effective models for triangular lattice quantum antiferromagnets

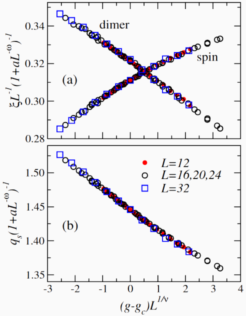
w/ Subir Sachdev, arXiv:2311.01572

Frustrated magnetism on non-bipartite lattices: a difficult problem

Bipartite lattices

Marshall sign rule allows for non-trivial

"designer Hamiltonians"

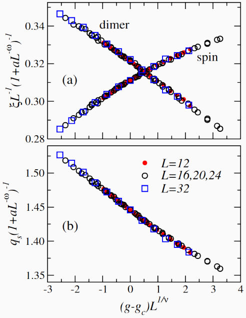


Sandvik, Phys. Rev. Lett. 98, 227202

Frustrated magnetism on non-bipartite lattices: a difficult problem

Bipartite lattices

Marshall sign rule allows for non-trivial "designer Hamiltonians"



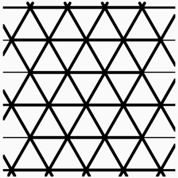
Sandvik, Phys. Rev. Lett. 98, 227202

Non-bipartite lattice

Primarily restricted to variational ansatzes (DMRG, PEPS, NQS. . .) or ED

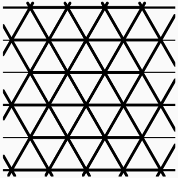
Effective models for triangular lattice quantum antiferromagnets

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



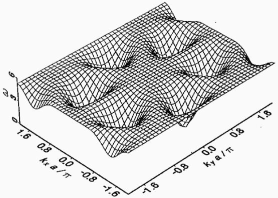
Effective models for triangular lattice quantum antiferromagnets

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

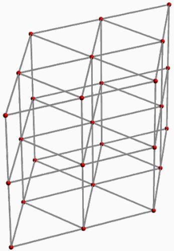


Effective model of bosonic spinons, U(1) gauge fluctuations Higgsed to Z_2

$$\vec{S}_i \equiv b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}$$

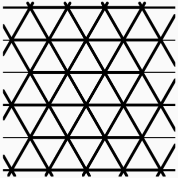


$$H = -\sum_{j,\mu,\alpha} J \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{c.c.} \right)$$



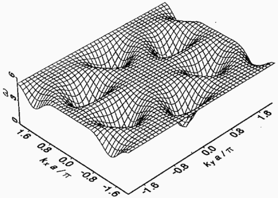
Effective models for triangular lattice quantum antiferromagnets

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

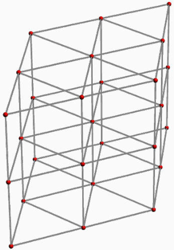


Effective model of bosonic spinons, U(1) gauge fluctuations Higgsed to Z_2

$$\vec{S}_i \equiv b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}$$



$$H = -\sum_{j,\mu,\alpha} J \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{c.c.} \right)$$



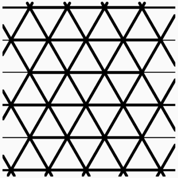
Couple to Z_2 gauge field, mutual statistics captured by Berry phase

$$H = -J \sum_{j,\mu,\alpha} s_{j,j+\hat{\mu}} \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{c.c.} \right)$$

$$-K \sum_{\triangle,\square} \prod_{\triangle,\square} s_{j,j+\hat{\mu}} + i\pi \sum_j s_{j,j+\hat{\tau}}$$

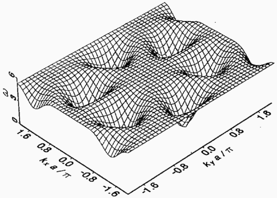
Effective models for triangular lattice quantum antiferromagnets

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Effective model of bosonic spinons, U(1) gauge fluctuations Higgsed to Z_2

$$\vec{S}_i \equiv b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}$$



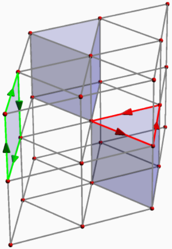
$$H = -\sum_{j,\mu,\alpha} J \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{c.c.} \right)$$

Couple to Z_2 gauge field, mutual statistics captured by Berry phase

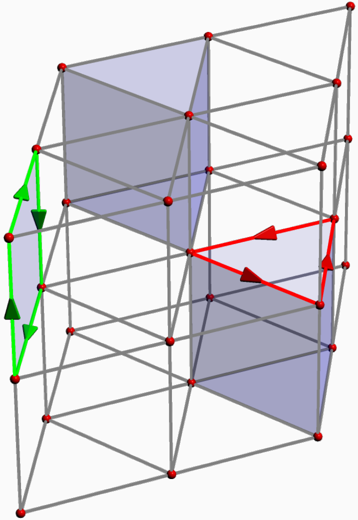
$$H = -J \sum_{j,\mu,\alpha} s_{j,j+\hat{\mu}} \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{c.c.} \right)$$

$$-K \sum_{\triangle,\square} \prod_{\triangle,\square} s_{j,j+\hat{\mu}} + i\pi \sum_j s_{j,j+\hat{\tau}}$$

Exact sign-problem free mapping, preserves emergent O(4) symmetry



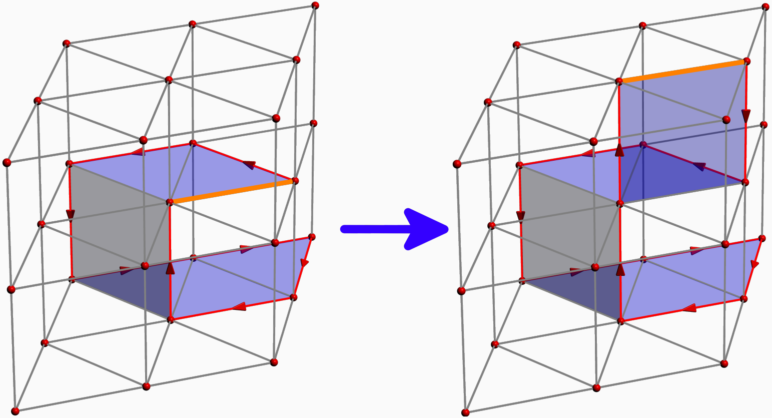
Duality transformation for bosons coupled to $\mathbb{Z}_2$ gauge fields



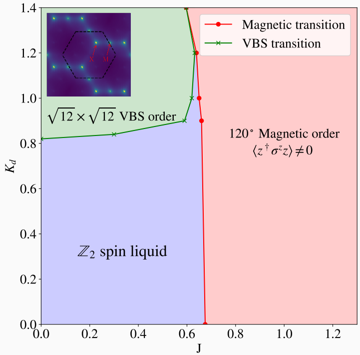
- Generalization of bosonic "world-lines" - odd world-lines must contain surfaces of gauge flux
- Berry phase contributes frustration in the surface action
- AF order = current proliferation, asymmetry in different current flavors

Worm algorithms difficult with gauge fluctuations

"Surface worm algorithm" allows for growth of ligaments, but is insufficient for avoiding diverging correlation time

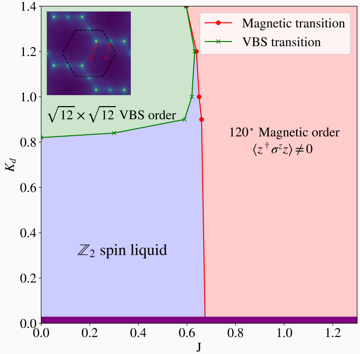


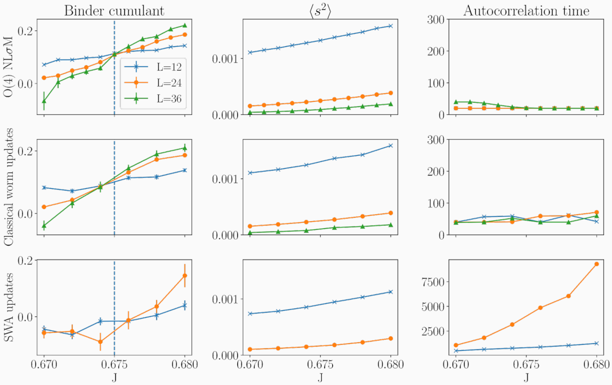
Monte Carlo simulations establish AF, VBS, and spin liquid phases



- VBS order only commensurate with system sizes multiples of 12
- Surprisingly technical simulation - geometrically complex and no "obvious" bottleneck
- Wolff cluster update utilized on gauge DOFs in addition to SWA

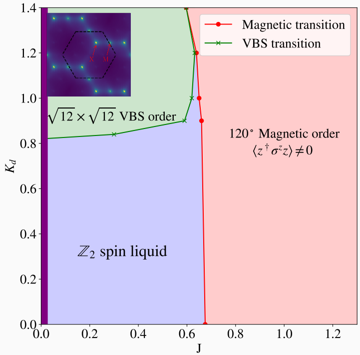
Monte Carlo simulations establish AF, VBS, and spin liquid phases

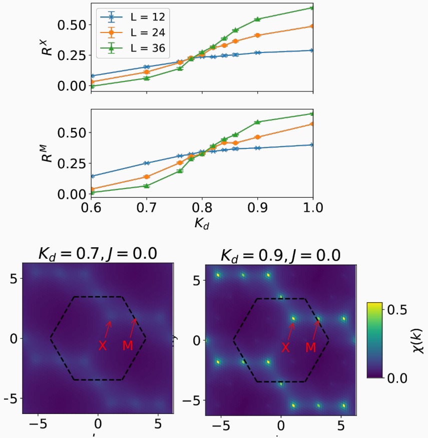




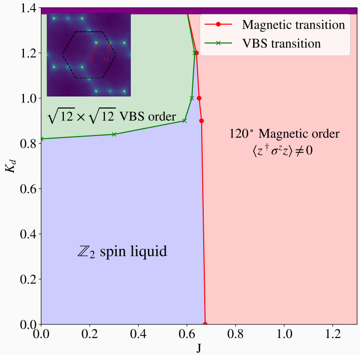
SWA still identifies transition, although restricted to small systems

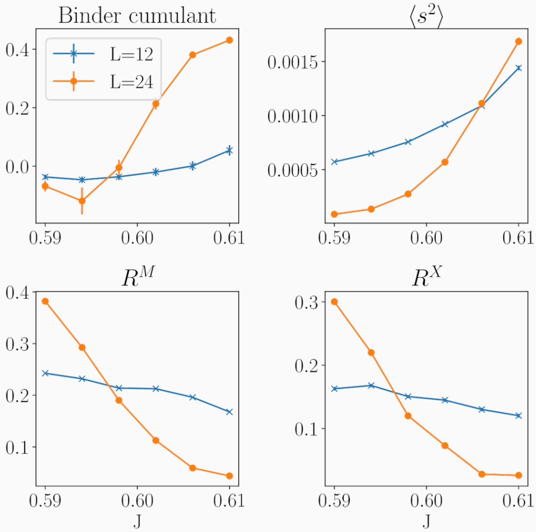
Monte Carlo simulations establish AF, VBS, and spin liquid phases





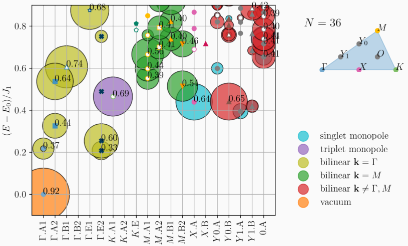
Monte Carlo simulations establish AF, VBS, and spin liquid phases

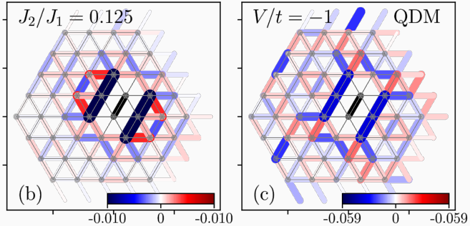




Applications to Heisenberg models

Low-energy spectrum of $J_1 - J_2$ model has high overlap with Dirac spin liquid and $\sqrt{12} \times \sqrt{12}$ VBS (Wietek, arXiv:2303.01585)

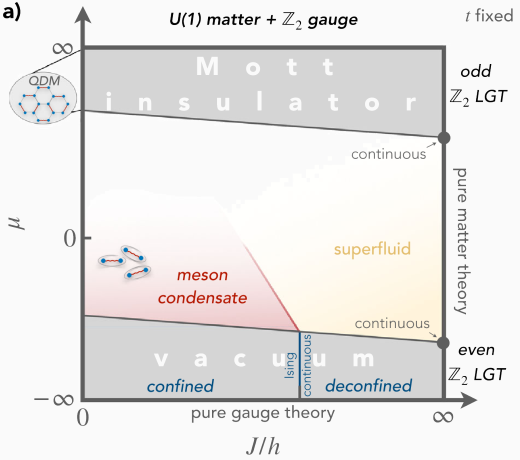




AF to VBS transition described by Dirac spin liquid (Jian, Phys. Rev. B 97, 195115)

Outlook and future directions

- Bosons coupled to discrete gauge fields remains a relatively unexplored research direction, also relevant for quantum simulators (Homeier et al. Commun Phys 6, 127 (2023)
- PIMC formulation is rather rudimentary, can this mapping be applied to continuous time? SSE?



Conductance and thermopower fluctuations in interacting quantum dots

Conductance and thermopower fluctuations in interacting quantum dots



w/ Laurel Anderson, Philip Kim, and Subir Sachdev, arXiv:2309.05741

SYK as a minimal model for holographic physics

$$H = \frac{1}{(2N)^{\frac{3}{2}}} \sum_{ijkl} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{N^{\frac{1}{2}}} \sum_{ij} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

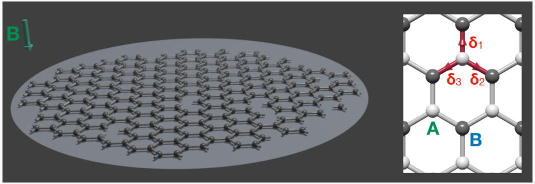
$$\langle J_{ij;kl} \rangle = \langle t_{ij} \rangle = 0 \quad \langle J^*_{ij;kl} J_{ij;kl} \rangle = J^2 \quad \langle t^*_{ij} t_{ij} \rangle = t^2$$

SYK as a minimal model for holographic physics

$$H = \frac{1}{(2N)^{\frac{3}{2}}} \sum_{ijkl} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{N^{\frac{1}{2}}} \sum_{ij} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$\langle J_{ij;kl} \rangle = \langle t_{ij} \rangle = 0 \quad \langle J^*_{ij;kl} J_{ij;kl} \rangle = J^2 \quad \langle t^*_{ij} t_{ij} \rangle = t^2$$

Proposed realizations in disordered graphene (Phys. Rev. Lett. 121, 036403)

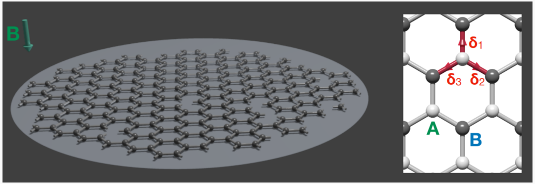


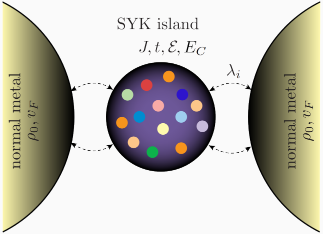
SYK as a minimal model for holographic physics

$$H = \frac{1}{(2N)^{\frac{3}{2}}} \sum_{ijkl} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{N^{\frac{1}{2}}} \sum_{ij} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

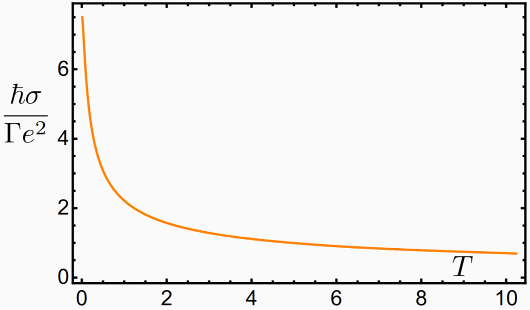
$$\langle J_{ij;kl} \rangle = \langle t_{ij} \rangle = 0 \quad \langle J^*_{ij;kl} J_{ij;kl} \rangle = J^2 \quad \langle t^*_{ij} t_{ij} \rangle = t^2$$

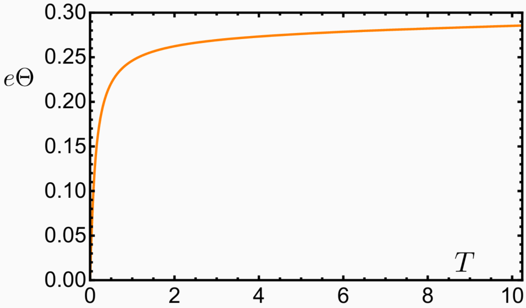
Proposed realizations in disordered graphene (Phys. Rev. Lett. 121, 036403)





Transport quantities: disordered Fermi liquid below $E_{\text{coh}} \sim t^2/J$, SYK above



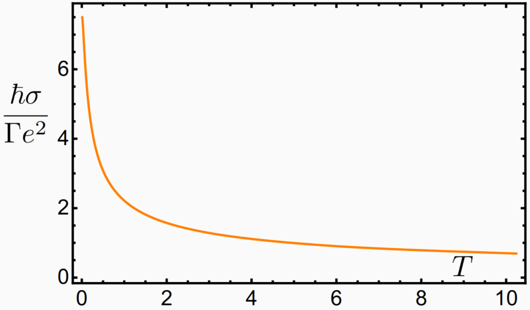


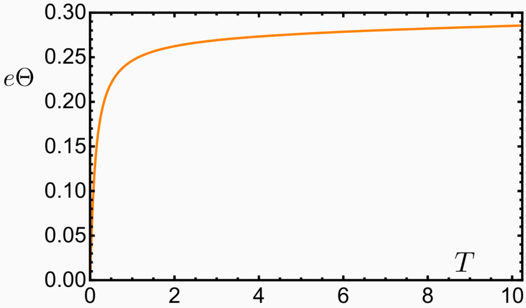
$$\sigma = \frac{4e^2\Gamma}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}\omega\, f'(\omega)\,\mathrm{Im}\,G(\omega)$$

$$\Theta = \frac{\beta}{e}\frac{\int_{-\infty}^{\infty} \mathrm{d}\omega\,\omega f'(\omega)\,\mathrm{Im}\,G(\omega)}{\int_{-\infty}^{\infty} \mathrm{d}\omega\, f'(\omega)\,\mathrm{Im}\,G(\omega)}$$

Phys. Rev. B 101, 205148 (2020)

Transport quantities: disordered Fermi liquid below $E_{coh} \sim t^2/J$, SYK above





$$\sigma = \frac{4e^2\Gamma}{\hbar} \int_{-\infty}^{\infty} \mathrm{d}\omega \, f'(\omega) \, \mathrm{Im}\, G(\omega)$$

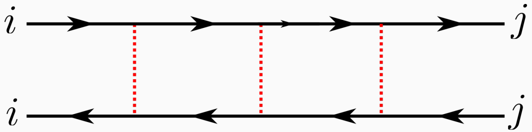
$$\Theta = \frac{\beta}{e} \frac{\int_{-\infty}^{\infty} \mathrm{d}\omega \, \omega f'(\omega) \, \mathrm{Im}\, G(\omega)}{\int_{-\infty}^{\infty} \mathrm{d}\omega \, f'(\omega) \, \mathrm{Im}\, G(\omega)}$$

Phys. Rev. B 101, 205148 (2020)

Can statistical fluctuations be used as a probe for strongly-correlated physics?

Non-interacting Fermi liquid prediction

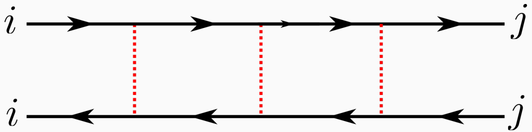
Key quantity to calculate: $\overline{\langle G_{ij}(i\omega)\rangle\langle G_{ji}(i\epsilon)\rangle}$



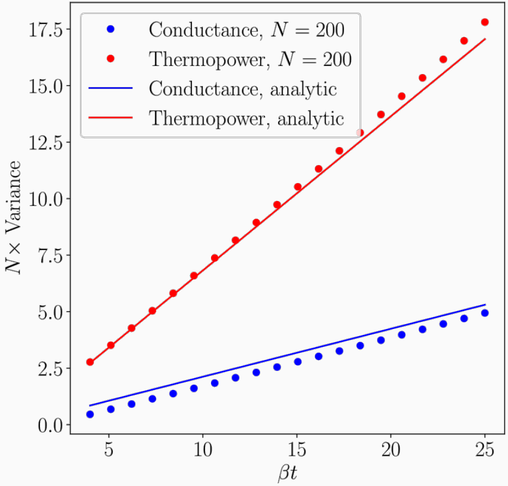
$$\text{Var } \sigma \sim \text{Var } \Theta \sim \frac{1}{NT}$$

Non-interacting Fermi liquid prediction

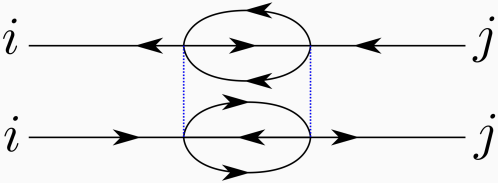
Key quantity to calculate: $\overline{\langle G_{ij}(i\omega)\rangle\langle G_{ji}(i\epsilon)\rangle}$



$$\text{Var } \sigma \sim \text{Var } \Theta \sim \frac{1}{NT}$$



Pure SYK prediction

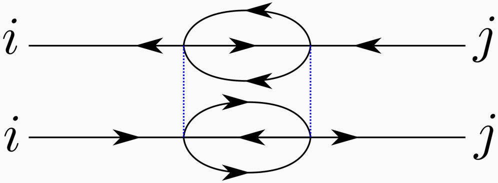


"Universal" fluctuations in conformal limit,

$$\frac{\text{Var } \sigma}{\sigma^2} = \frac{2}{N^3}$$

$$\text{Var } \Theta = \mathcal{O}\left(N^{-4}\right)$$

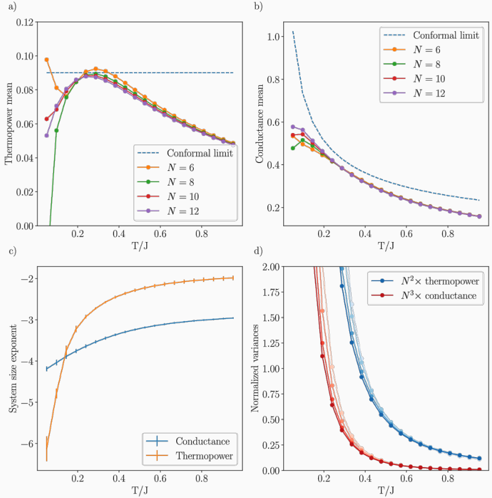
Pure SYK prediction



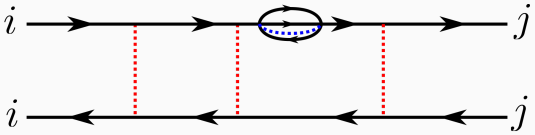
"Universal" fluctuations in conformal limit,

$$\frac{\text{Var } \sigma}{\sigma^2} = \frac{2}{N^3}$$

$$\text{Var } \Theta = \mathcal{O}(N^{-4})$$

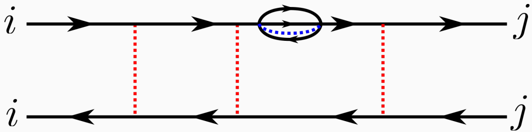


Random hoppings still drive fluctuations even in SYK regime!

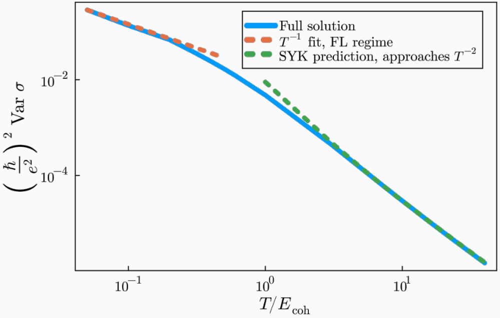


SYK interactions renormalize ladder propagators, fluctuations still remain $\mathcal{O}(N^{-1})$ for $T \gg E_{\mathrm{coh}}$

Random hoppings still drive fluctuations even in SYK regime!



SYK interactions renormalize ladder propagators, fluctuations still remain $\mathcal{O}(N^{-1})$ for $T \gg E_{\mathrm{coh}}$



T^{-1} to T^{-2} crossover signals SYK physics

Outlook

- These results worked within an *equilibrium* setting - can we do better? Recover UCF as $T \to 0$?
- Fluctuations for SYK in Schwarzian-dominated regime may yield new results