Models of deconfined criticality on square and triangular lattice antiferromagnets

Henry Shackleton

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Harvard University

Quantum magnetism as a platform for exotic phases

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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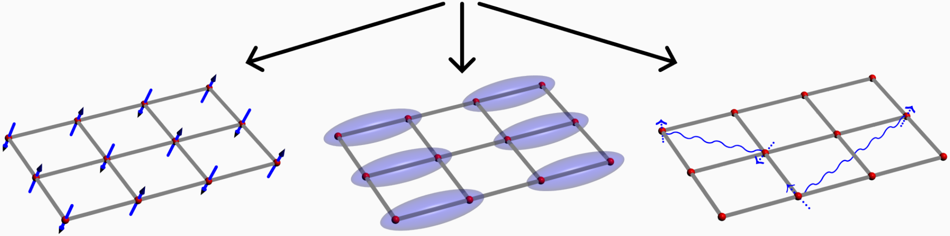

Magnetic order　　Valence bond solid order　　Quantum spin liquid

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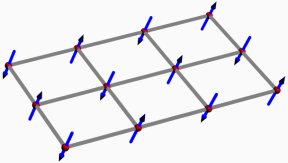
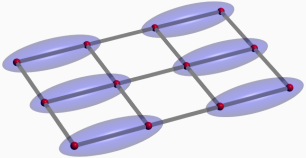
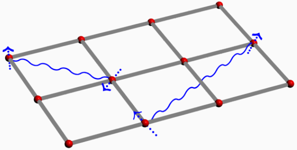


Magnetic order



Valence bond solid order



Quantum spin liquid

Parton construction: versatile theoretical tool

$$\vec{S}_i \to b^\dagger_{i\alpha} \vec{\sigma}^{\alpha\beta} b_{i\beta} \,,\, f^\dagger_{i\alpha} \vec{\sigma}^{\alpha\beta} f_{i\beta}$$

Quantum magnetism as a platform for exotic phases

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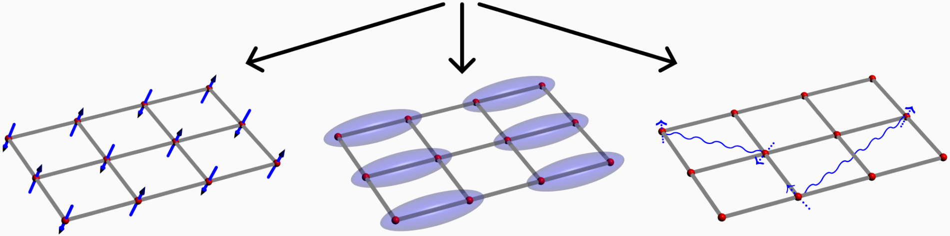


Magnetic order | Valence bond solid order | Quantum spin liquid

Square lattice: fermionic spinons for unifying numerically-observed Néel/spin liquid/VBS transitions

Triangular lattice: bosonic spinons for effective sign-problem-free model of triangular lattice DQCP

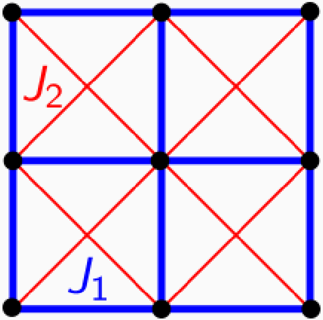
Deconfined criticality and a gapless $\mathbb{Z}_2$ spin liquid on the square lattice antiferromagnet

Deconfined criticality on the square lattice antiferromagnet

H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022 (7), 1-35

H. Shackleton, A. Thomson, S. Sachdev, Physical Review B 104 (4), 045110

Multimethod studies on $J_1 - J_2$ model indicate spin liquid phase



$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

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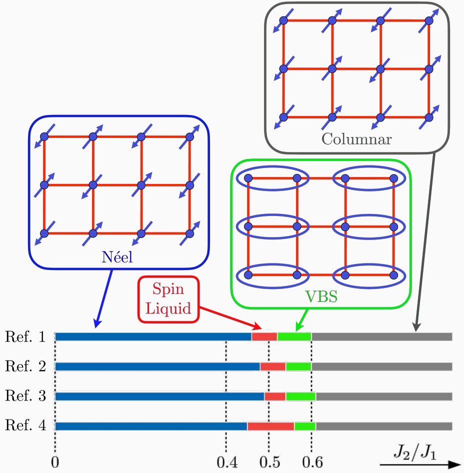

[1]Wang and Sandvik, *Phys. Rev. Lett.*, 2018 [2]Ferrari and Becca, *Phys. Rev. B*,. 2020, [3]Nomura and Imada, *Phys. Rev. X*,. 2021 [4]Liu et al., *Phys. Rev. X*,. 2022

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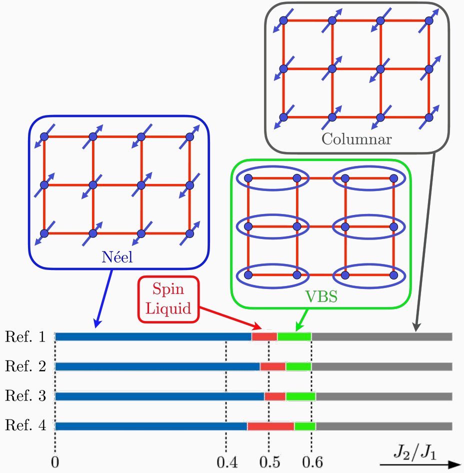
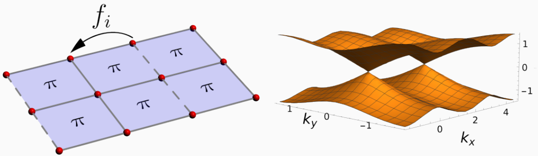


Assume VMC description of spin liquid, gapless fermionic spinons with d-wave pairing (Z2Azz13)

[1] Wang and Sandvik, *Phys. Rev. Lett.*, 2018 [2] Ferrari and Becca, *Phys. Rev. B*,. 2020, [3] Nomura and Imada, *Phys. Rev. X*,. 2021 [4] Liu et al., *Phys. Rev. X*,. 2022

π-flux as a "parent" phase of a $\mathbb{Z}_2$ spin liquid



N_f=2 QCD_3, emergent SO(5) symmetry

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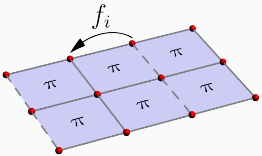

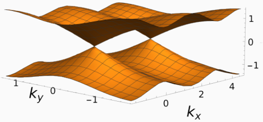

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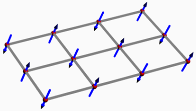
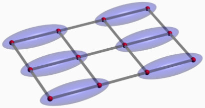
Unstable to
Neel/VBS order





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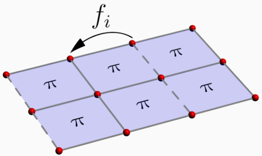

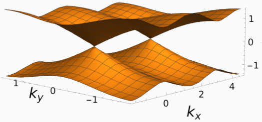

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Higgs Φ_1^a , Φ_2^a

$SU(2) \to \mathbb{Z}_2$

Z2Azz13

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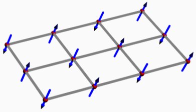
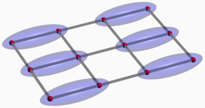
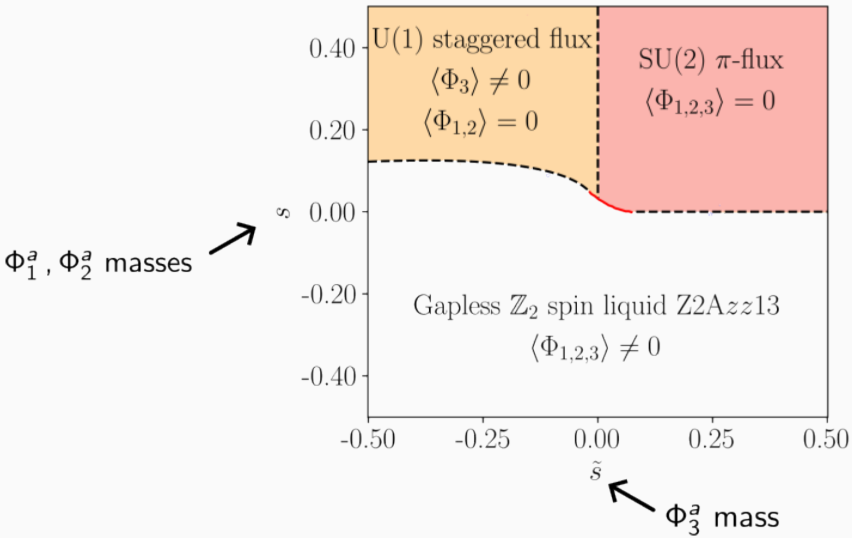




Multiple instabilities captured by proximity to Dirac spin liquid



$\mathrm{U}(1) \to \mathbb{Z}_2$ transition has fixed spinon anisotropy

[6] Hermele, Senthil, and Fisher, *Phys. Rev. B*,. 2005

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Pure QED_3: fermion anisotropy irrelevant, emergent Lorentz symmetry [6]

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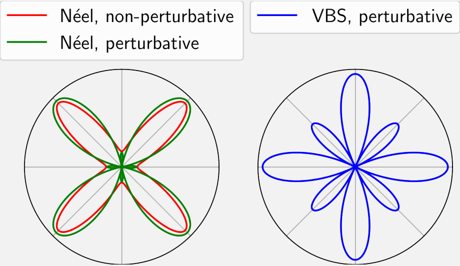
$$\gamma^\mu k_\mu \pm \Phi(\gamma^y k_x + \gamma^x k_y)$$

$$\Phi_c \approx 0.458 + \mathcal{O}\left(N_f^{-1}\right)$$



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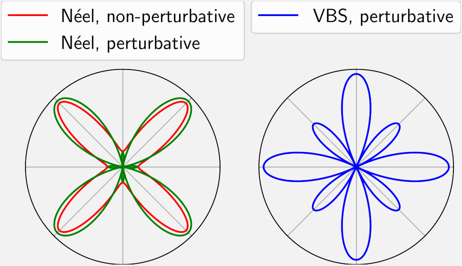
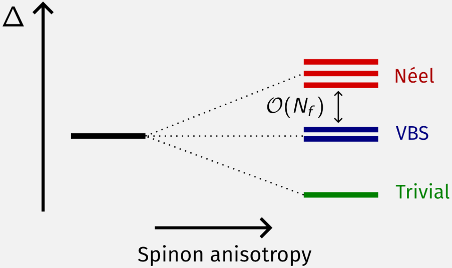


$\eta_{\text{Néel}} \sim \eta_{\text{VBS}}$, but monopole splitting may be more relevant



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UV/IR mixing in $\mathrm{SU}(2) \to \mathbb{Z}_2$ transition

$$\mathcal{L} = \mathcal{L}_{N_f=2\ \mathrm{QCD}_3} + \lambda \left(\Phi_1^a \bar{\psi} \gamma^x \mu^z \sigma^a \psi + \Phi_2^a \bar{\psi} \gamma^y \mu^x \sigma^a \psi \right)$$

Valley (μ^z) Gauge (σ^a)

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Conserved "currents"

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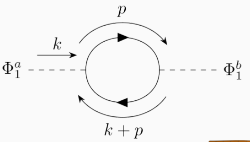
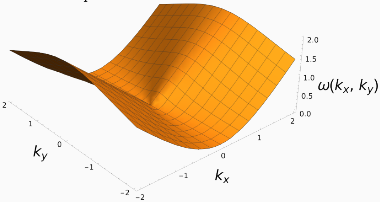


$$\sim \frac{k_0^2 + k_y^2}{\sqrt{k^2}}$$

Emergent "Higgs Bose liquid," extensive gapless modes regulated by (irrelevant) $\Phi\partial^2\Phi$ term



UV/IR mixing in SU(2) → $\mathbb{Z}_2$ transition

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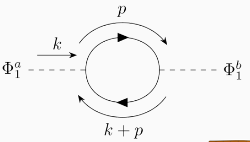

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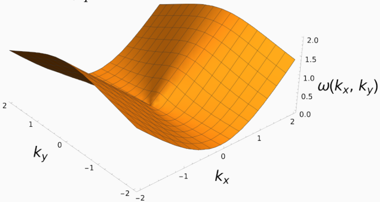
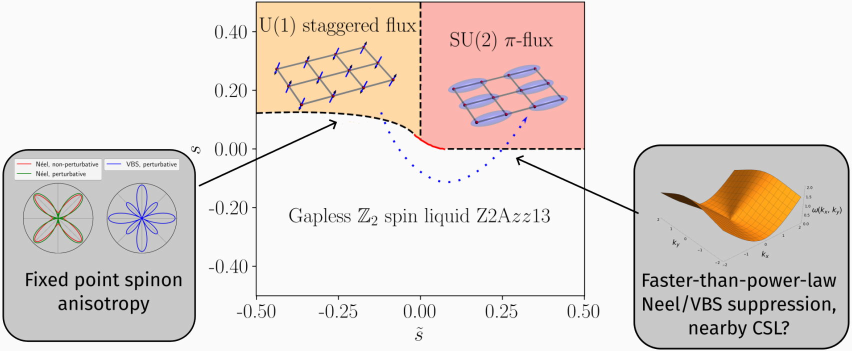


$$G_{\text{Néel}}(r) \sim \exp\left[-\eta_{\text{Néel}} \ln^2(r/a)\right]$$

$$G_{\text{VBS}}(r) \sim \exp\left[-\eta_{\text{VBS}} \ln^2(r/a)\right]$$

$$\eta_{\text{Néel}} > \eta_{\text{VBS}}$$

Summary and outlook



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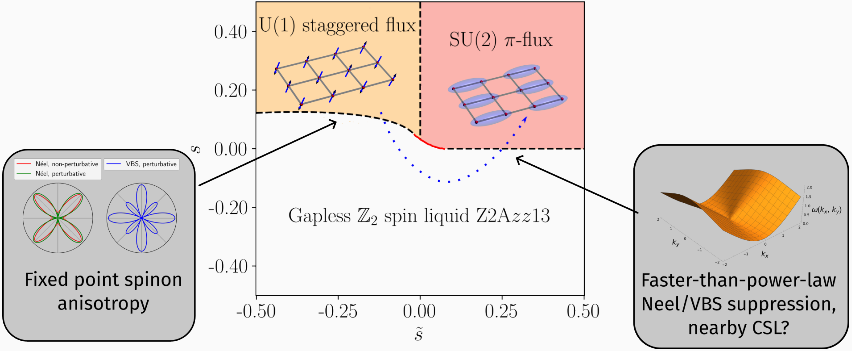

- Are $\log^2$ predictions accurate? Can we find a minimal model? With numerics?

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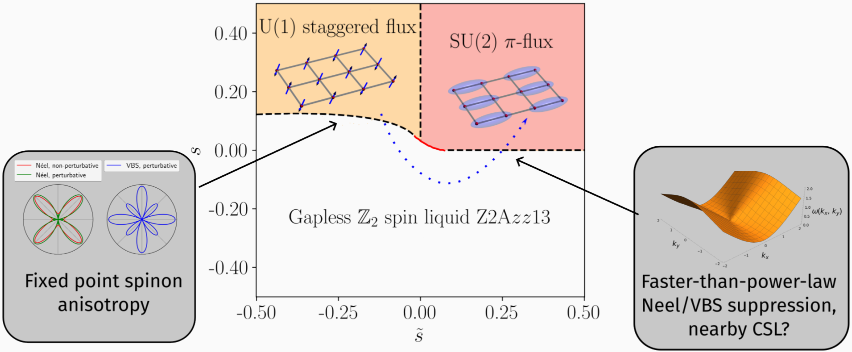

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- Similar ideas in engineering NFLs[7], Thirring models[8]...

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Sign-problem-free effective models for triangular lattice quantum antiferromagnets

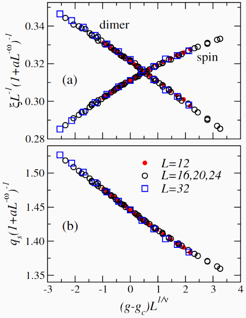
Effective models for triangular lattice quantum antiferromagnets

H. Shackleton and S. Sachdev, arXiv:2311.01572

Frustrated magnetism on non-bipartite lattices: a difficult problem

Bipartite lattices

Marshall sign rule allows for non-trivial "designer Hamiltonians" [9]



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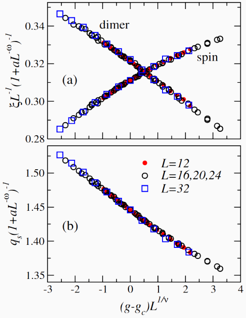

Non-bipartite lattice

Primarily restricted to variational ansatzes (DMRG, PEPS, NQS. . .) or ED

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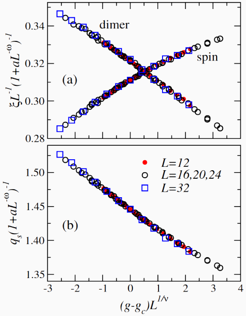

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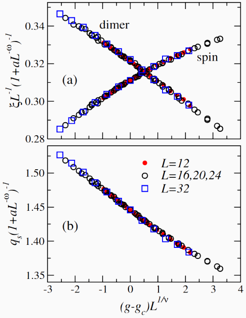

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Goal: construct an effective model amenable to large-scale QMC simulations

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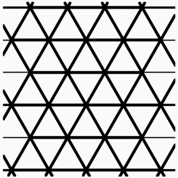
Effective models for triangular lattice quantum antiferromagnets

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



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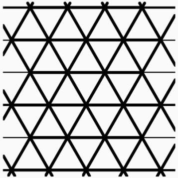
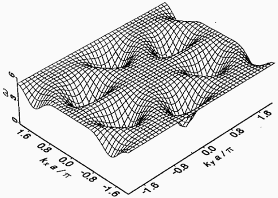


Effective model of bosonic spinons, U(1) gauge fluctuations Higgsed to Z_2

$$\vec{S}_i \equiv b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}$$



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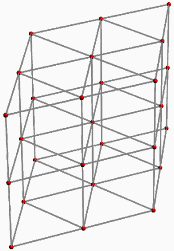

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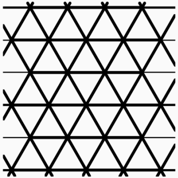

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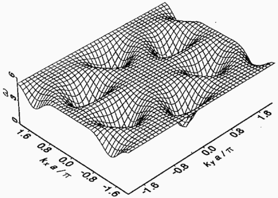
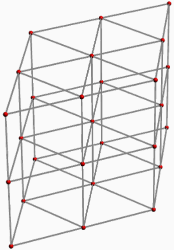


$$H = -\sum_{j,\mu,\alpha} J \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{c.c.} \right)$$

Couple to Z_2 gauge field, mutual statistics captured by Berry phase



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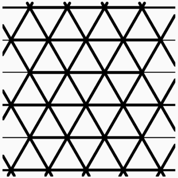

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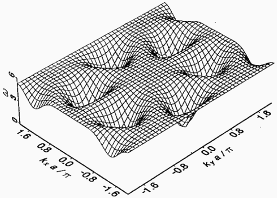

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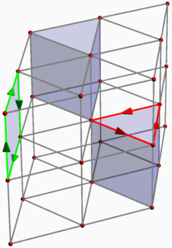
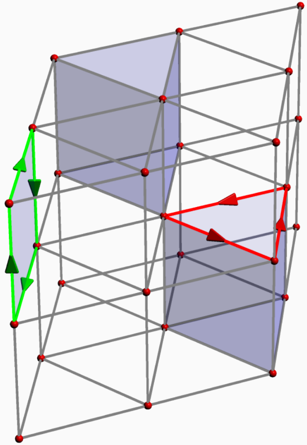
Exact sign-problem free mapping, preserves emergent O(4) symmetry



Duality transformation for bosons coupled to $\mathbb{Z}_2$ gauge fields



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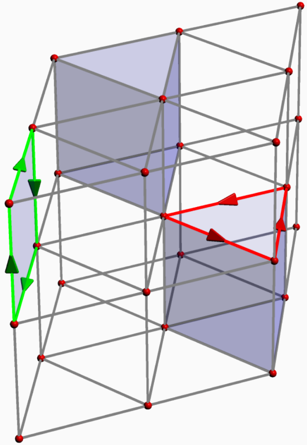
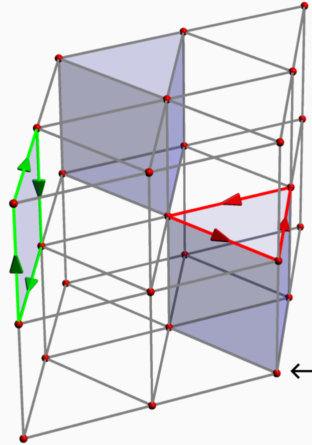


Bosonic "worldlines"
Two flavors for O(4)

Duality transformation for bosons coupled to $\mathbb{Z}_2$ gauge fields

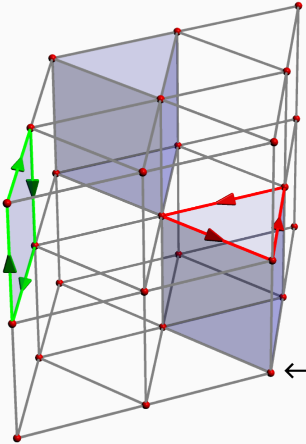


Bosonic "worldlines"
Two flavors for O(4)

Real-valued site DOF
Necessary to preserve O(4) exactly

Duality transformation for bosons coupled to $\mathbb{Z}_2$ gauge fields

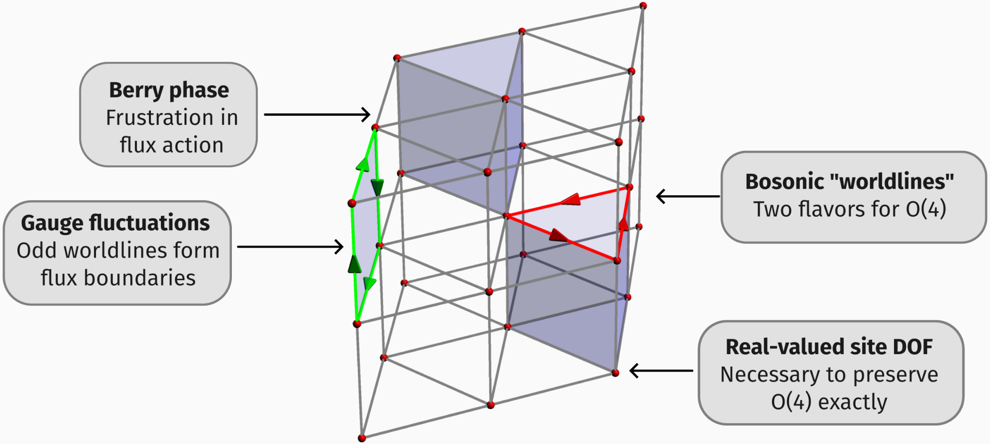
Gauge fluctuations
Odd worldlines form flux boundaries



Bosonic "worldlines"
Two flavors for O(4)

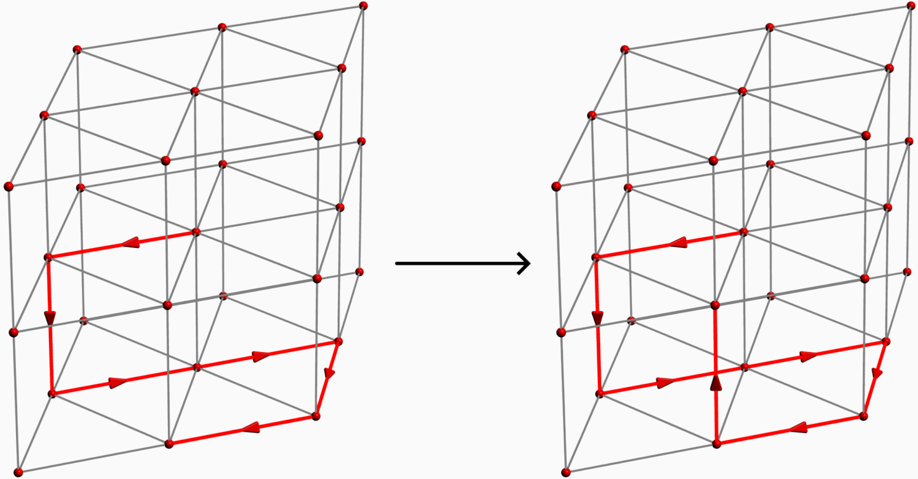
Real-valued site DOF
Necessary to preserve O(4) exactly

Duality transformation for bosons coupled to $\mathbb{Z}_2$ gauge fields



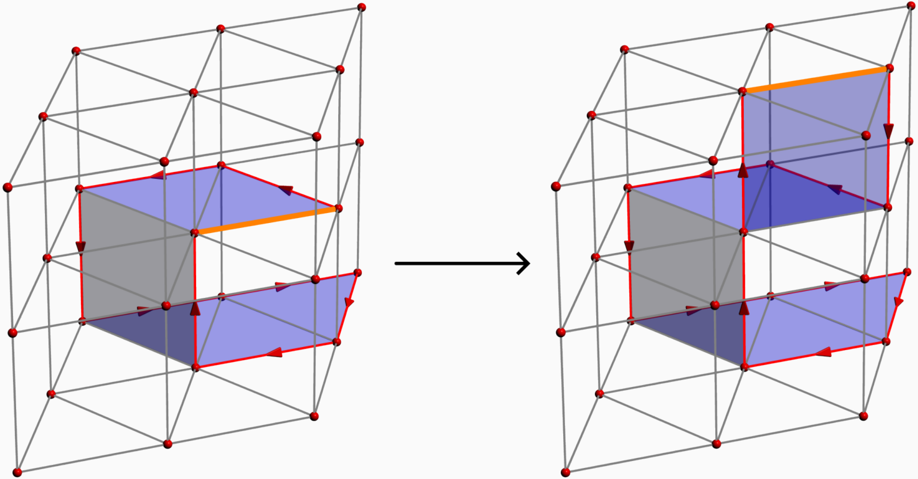
Worm algorithms difficult with gauge fluctuations

"Classical worm algorithm" effective without gauge fluctuations

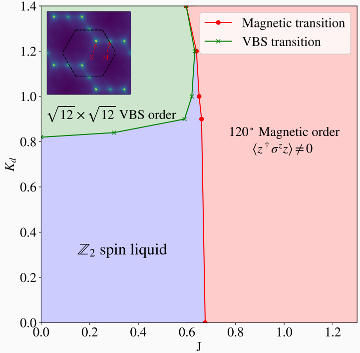


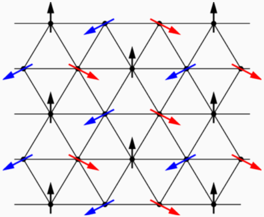
Worm algorithms difficult with gauge fluctuations

"Surface worm algorithm" works well but still has diverging AC

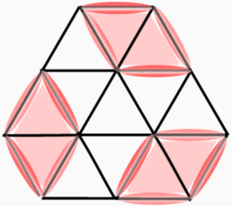


Monte Carlo simulations establish AF, VBS, and spin liquid phases



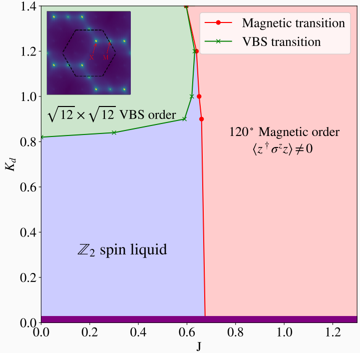


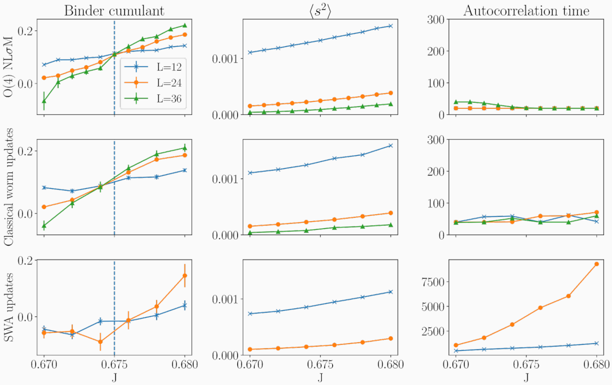
Magnetic order
Current loop proliferation, generically asymmetric



VBS order
Trans. symmetry breaking of flux configurations

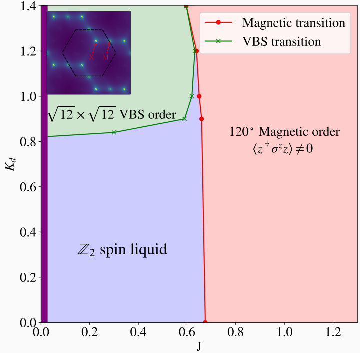
Monte Carlo simulations establish AF, VBS, and spin liquid phases

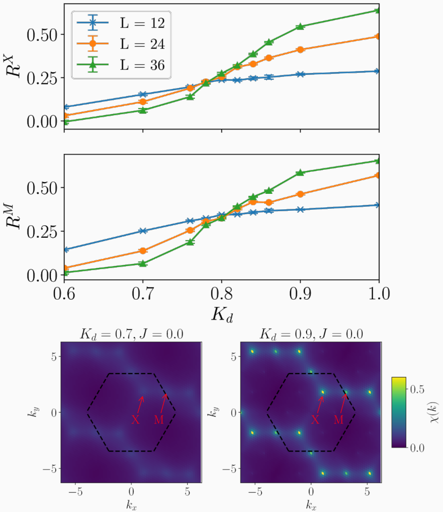




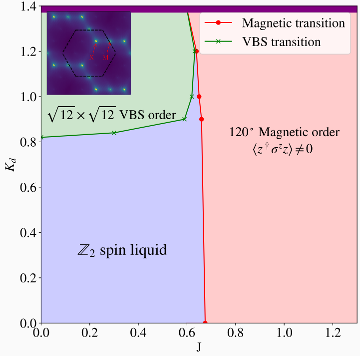
SWA still identifies transition, although restricted to small systems

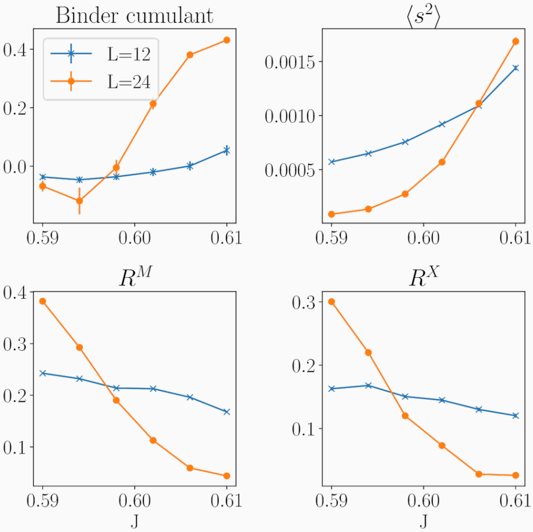
Monte Carlo simulations establish AF, VBS, and spin liquid phases





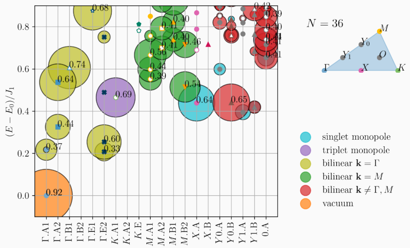
Monte Carlo simulations establish AF, VBS, and spin liquid phases

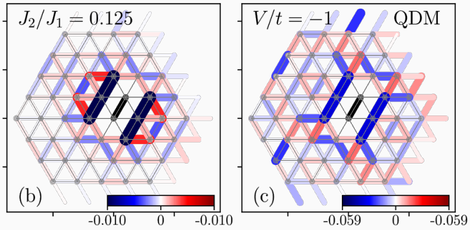




Applications to Heisenberg models

Low-energy spectrum of $J_1 - J_2$ model has high overlap with Dirac spin liquid and $\sqrt{12} \times \sqrt{12}$ VBS[11]

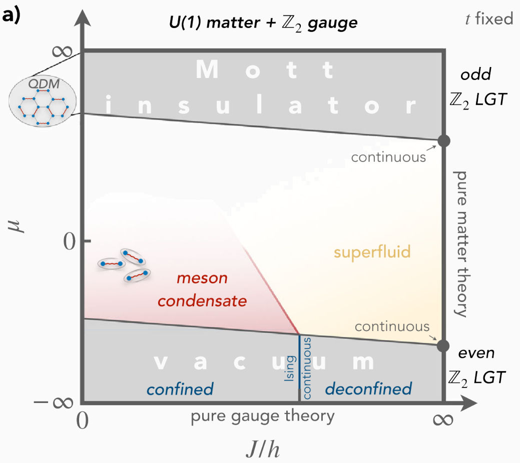




[11] Wietek, Capponi, and Läuchli, *arXiv e-prints*, 2023.

Outlook and future directions

- Bosons coupled to discrete gauge fields remains a relatively unexplored research direction, also relevant for quantum simulators [12]
- PIMC formulation is rather rudimentary, can this mapping be applied to continuous time? SSE?



[12] Homeier et al., *Commun. Phys.*, 2023